# Phase diagram and Hosotani mechanism in QCD-like theory with compact dimensions

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# Introduction

Understanding of spontaneous gauge symmetry breaking is important key in phyics beyond the standard model.

Hosotani mechanism : Y. Hosotani, Phys. Lett. B 126 (1983) 309; Ann. Phys. 190 (1989) 233.

When the extra dimension is **not simply connected**,
the extra-dimensional gauge field component can have the **nontrivial vacuum expectation value**.

Wilson loop in compacted direction

Eigen value

$$W = P\exp\left\{ig\int_C dy A_y\right\} \longrightarrow \mathrm{diag}[e^{2\pi i q_1}, e^{2\pi i q_2}, \cdots, e^{2\pi i q_N}]$$

$$q_i \neq q_j \Rightarrow m_n^2 = \frac{1}{L^2}(n + q_i - q_j)^2$$

Gauge symmetry breaking is happen

For example, $q_1 = q_2 \neq q_3$ : SU(2) × U(1) , $q_1 \neq q_2 \neq q_3$ : U(1) × U(1)

Nontrivial *q* breaks the gauge symmetry and then **Higgs can be considered as a fluctuation of $A_y$**.

ex) K. Agashe, R. Contino and A. Pomarol, Nucl. Phys. B 719 (2005) 165, S. Funatsu, H. Hatanaka, Y. Hosotani, Y. Orikasa, T. Simotani, Phys. Lett. B 722 (2013) 94.

Promising model: $SO(5) \times U(1) \xrightarrow{\text{Orbifold boundary condition}} SO(4) \times U(1) \xrightarrow{\text{Braine dynamics}} SU(2) \times U(1) \xrightarrow{\text{Hosotani mechanism}} U(1)$

The gauge symmetry breaking have been investigated in **Quantum chromodynamic**.

**Adjoint fermion**

M. Unsal, Phys. Rev. Lett. 100 (2008) 032005, J. C. Myers, M. C. Ogilvie, Phys. Rev. D 77 (2008) 125030.
G. Cossu, M. D'Elia, JHEP 07(2009) 048.

Adjoint fermion with periodic boundary condition leads the spontaneous gauge symmetry breaking.

**Fundamental fermion**

The fundamental fermion usually can not lead the gauge symmetry breaking.

But, it is possible by using **flavor twisted boundary condition**.

H. Kouno, Y. Sakai, T. Makiyama, K. Tokunaga, T. Sasaki, and M. Yahiro,
J. Phys. G: Nucl. Part. Phys. **39** (2012) 085010.

H. Kouno, T. Misumi, **K.K.**, T. Makiyama, T. Sasaki, M. Yahiro, Phys. Rev .D 88 (2013) 016002.

In both cases, the boundary condition of fermion plays a important role.

In this talk, we mainly discuss the gauge symmetry breaking in 4D. This system is good laboratory to understand the Hosotani mechanism.

Some our results in 5D are shown in K.K. and T. Misumi, JHEP 05 (2013) 042.

# Imaginary chemical potential and boundary condition

Recently, effects of the boundary condition of fermion are energetically investigated in different context.

The **imaginary chemical potential** appeared in QCD is important. A. Roberge and N. Weiss, Nucl. Phys. B275 (1986) 734.

Matsubara frequency with the imaginary chemical potential

Imaginary chemical potential

Fermion : $\omega_n^f = 2\pi T\,(n + 1/2) + \mu_I$

It comes from the anti-periodic boundary condition.

Imaginary chemical potential can be transformed to the boundary angle

$$\omega_n^f = 2\pi T\,(n + \phi) \longrightarrow \omega_n^f = 2\pi T\,(n + 1/2) - \pi T + 2\pi T\phi$$

Boundary angle

It can be considered as the imaginary chemical potential

Therefore, we may use knowledge obtained in investigation of QCD phase diagram to spontaneous gauge symmetry breaking phenomena.

This study is also related with the investigation of QCD structure itself.

**Perturbative one-loop potential**

D. Gross, R. Pisarski, L. Yaffe, Rev. Mod. Phys 53 (1981) 43.
N. Weiss, Phys.Rev.D 24 (1981) 475.

Here we consider 4D.
Euclidean temporal direction is treated as compact dimension.
Arbitral dimensional representations can be obtained by the same way.

ex) Sakamoto and Takenaga, Phys. Rev. D80 (2009) 085016.

## Gauge boson

$$\mathcal{V}_g(q) = -\frac{2}{L^4\pi^2}\sum_{i,j=1}^{N}\sum_{n=1}^{\infty}\left(1-\frac{1}{N}\delta_{ij}\right)\frac{\cos[2n\pi q_{ij}]}{n^4}$$

## Fundamental fermion

$$\mathcal{V}_f^{\phi}(q; N_f, m_f) = \frac{2N_f m_f^2}{\pi^2 L^2}\sum_{i=1}^{N}\sum_{n=1}^{\infty}\frac{K_2(nm_f L)}{n^2}\cos[2\pi n(q_i+\phi)]$$

## Adjoint fermion

$$\mathcal{V}_a^{\phi}(q; N_a, m_a) = \frac{2N_a m_a^2}{\pi^2 L^2}\sum_{i,j=1}^{N}\sum_{n=1}^{\infty}\left(1-\frac{1}{N}\delta_{ij}\right)\frac{K_2(nm_a L)}{n^2}\cos[2\pi n(q_{ij}+\phi)]$$

## Perturbative one-loop potential

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### Fundamental fermion

Boundary angle: Fermion : 1/2, Boson : 0, π

$$\mathcal{V}_f^{\phi}(q;N_f,m_f) = \frac{2N_f m_f^2}{\pi^2 L^2}\sum_{i=1}^{N}\sum_{n=1}^{\infty}\frac{K_2(nm_fL)}{n^2}\cos[2\pi n(q_i+\phi)]$$

### Adjoint fermion

$$\mathcal{V}_a^{\phi}(q;N_a,m_a) = \frac{2N_a m_a^2}{\pi^2 L^2}\sum_{i,j=1}^{N}\sum_{n=1}^{\infty}\left(1-\frac{1}{N}\delta_{ij}\right)\frac{K_2(nm_aL)}{n^2}\cos[2\pi n(q_{ij}+\phi)]$$

Flavor number      Fermion mass

Phase : $\langle A_y\rangle = \frac{2\pi}{gL}q$

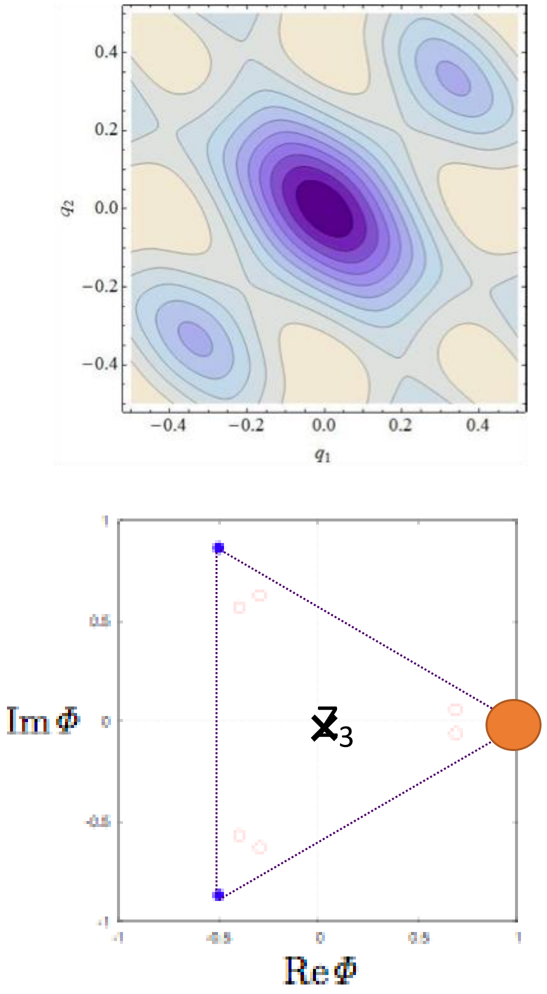
We introduce the fermion mass as a scale to control the gauge symmetry breaking.

# Contour plot without gauge symmetry breaking

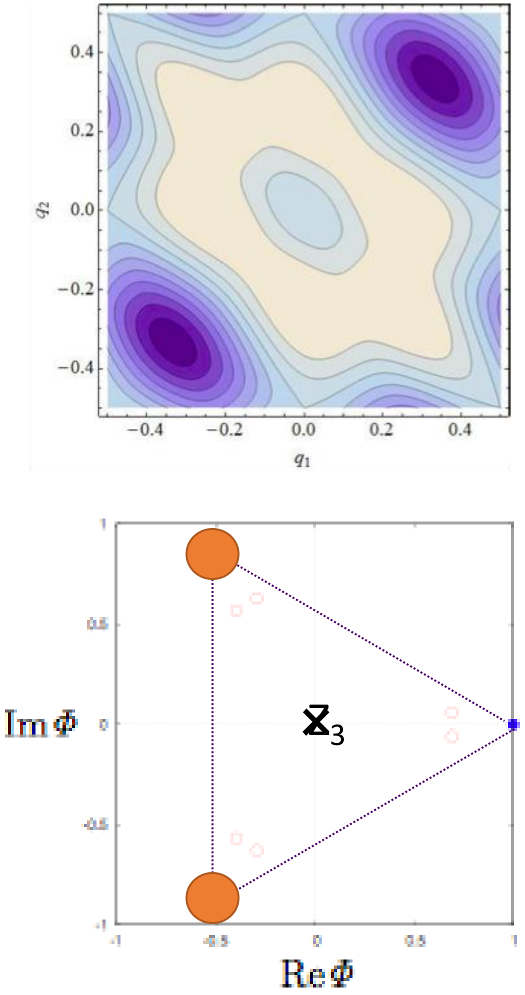
K.K. and T. Misumi, JHEP 05 (2013) 042

**Contour plot** $$q_1 + q_2 + q_3 = 0 \pmod 1$$ In these cases, there are no gauge symmetry breaking.

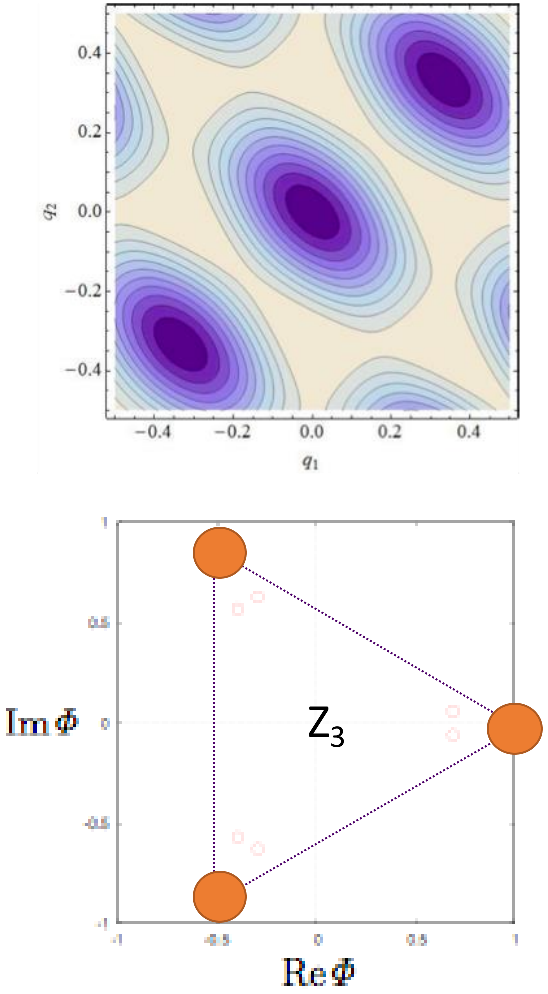
(1) aPBC fund.




(2) PBC fund.




(3) aPBC adjoint



## Contour plot with gauge symmetry breaking

K.K. and T. Misumi, JHEP 05 (2013) 042

**Gauge symmetry breaking**

$$q_1 + q_2 + q_3 = 0 \pmod{1}$$

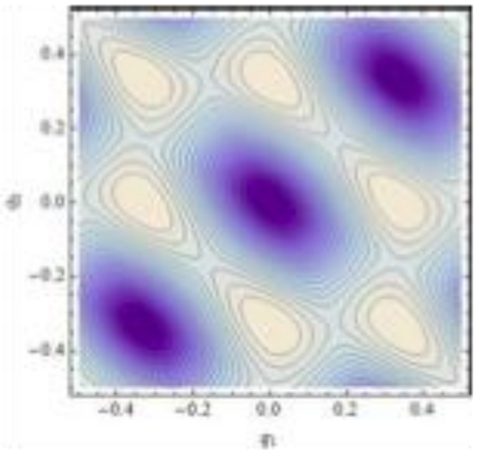
Gauge boson + adjoint fermion
with periodic boundary condition

SU(3) → Deconfined phase

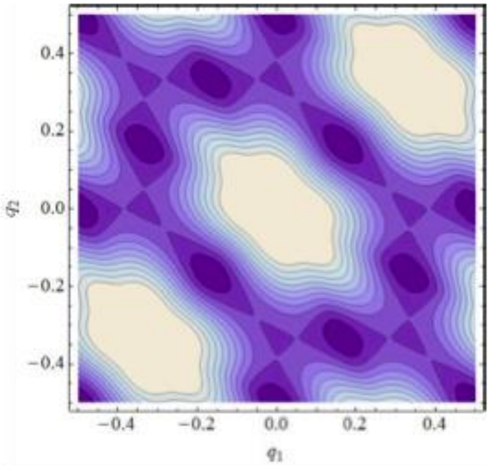
SU(2) × U(1) → Split (Skewed) phase

U(1) × U(1) → Re-confined phase

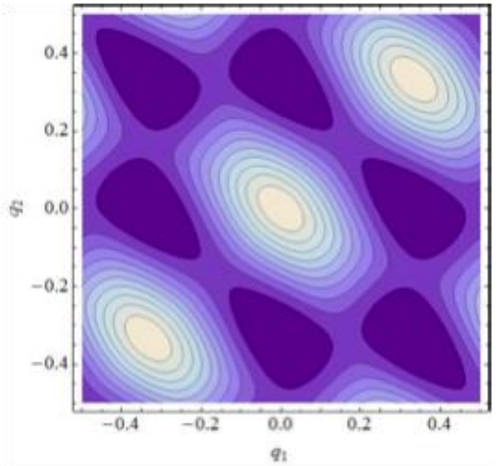
| $\boxed{q_1 = q_2 = q_3}$ | $\boxed{q_1 = q_2 \neq q_3}$ | $\boxed{q_1 \neq q_2 \neq q_3}$ |
|---|---|---|
| Large m | Medium m | Small m |



SU(3)




SU(2) × U(1)

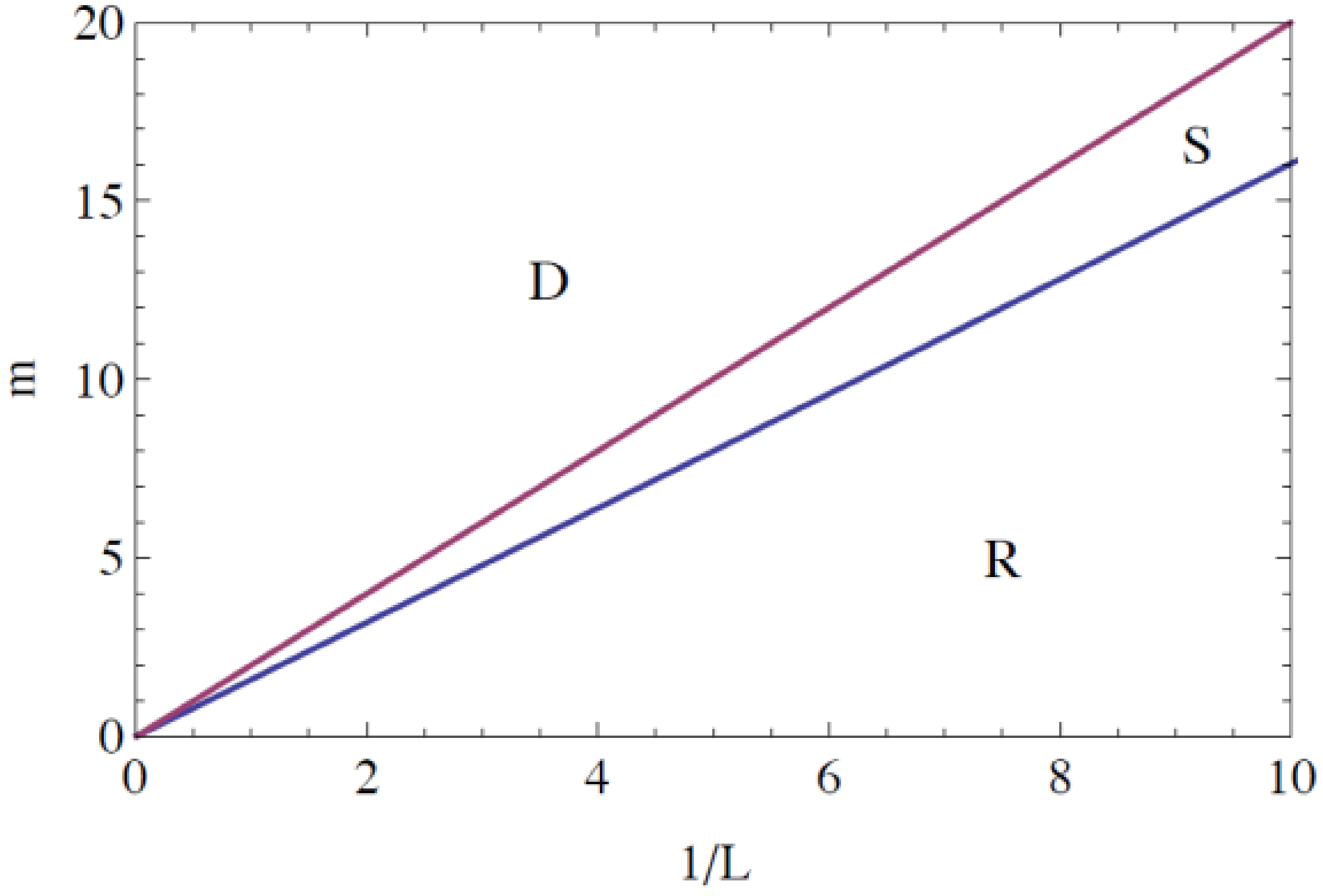



U(1) × U(1)

K.K. and T. Misumi, JHEP 05 (2013) 042

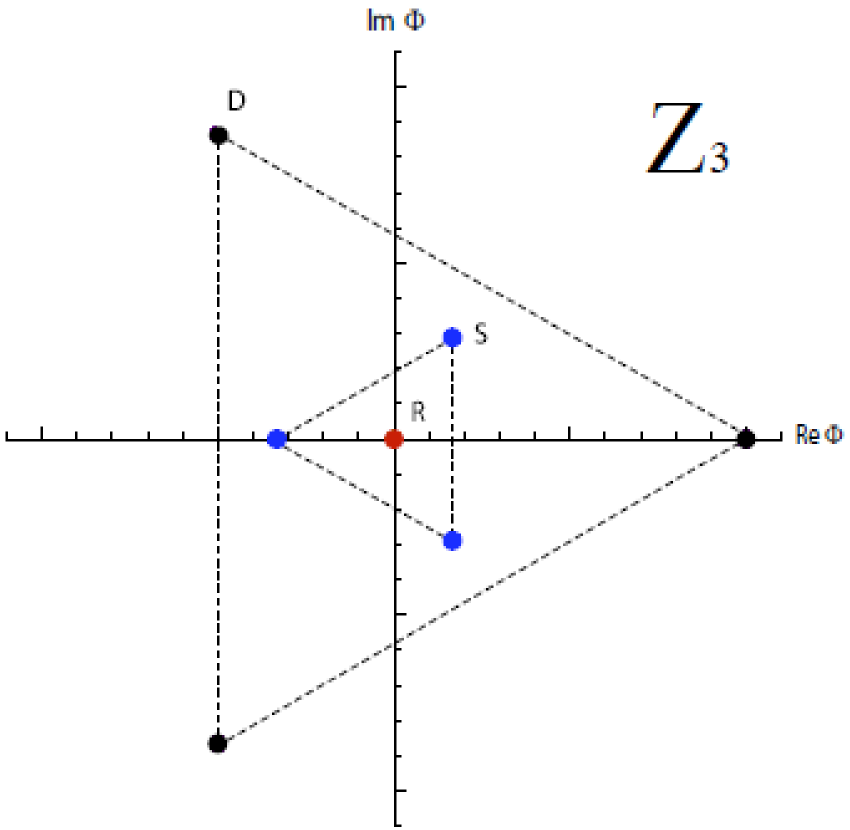
Phase Structure

Gauge boson + adjoint fermion
with periodic boundary condition




Similar phase diagram was obtained by using PNJL-type model
in H. Nishimura and M. Ogilvie, Phys. Rev. D 81 (2010) 014018.

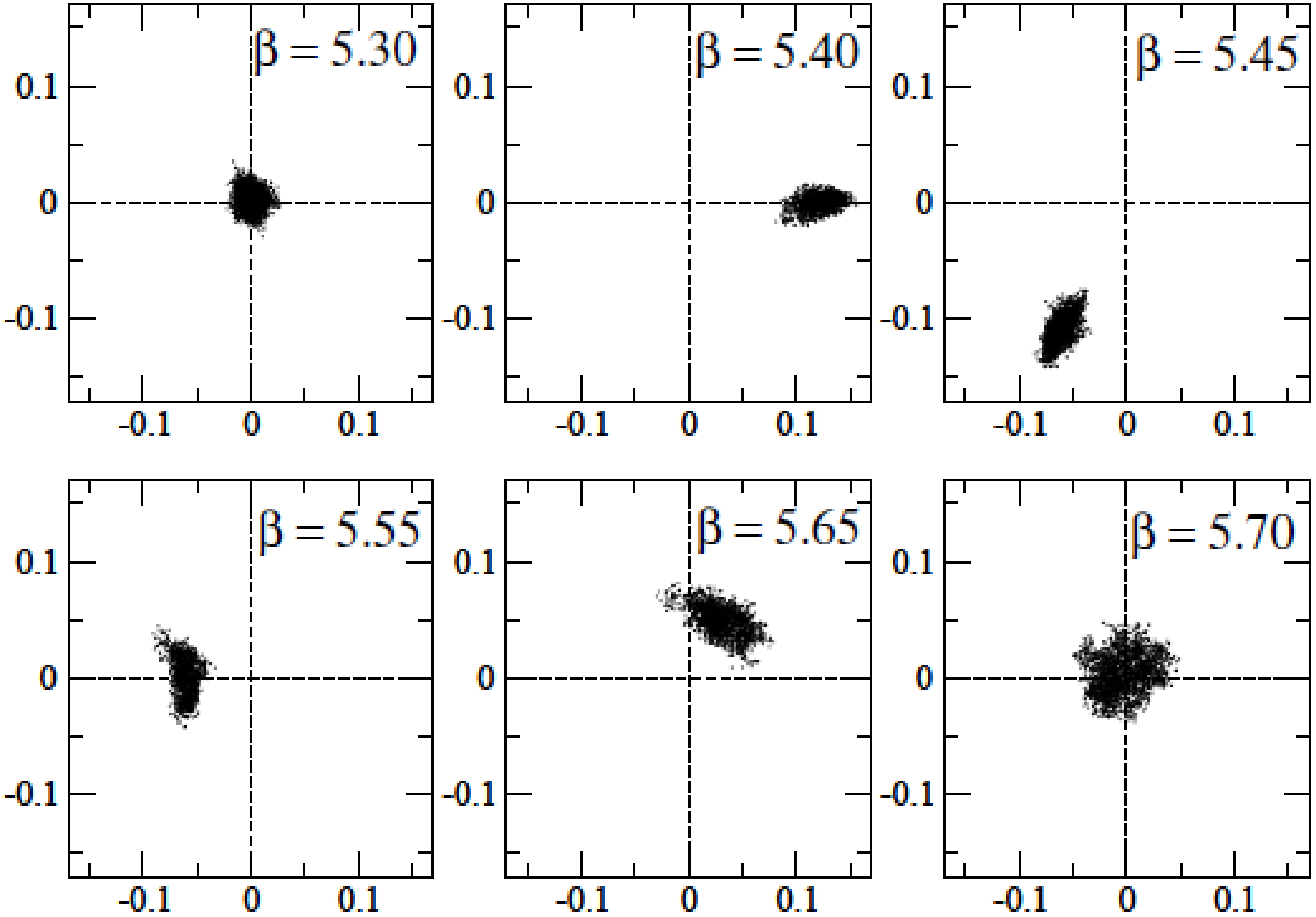
Distribution plot of Polyakov-loop



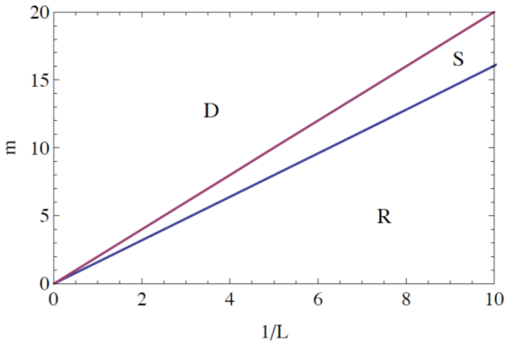
Hosotani mechanism

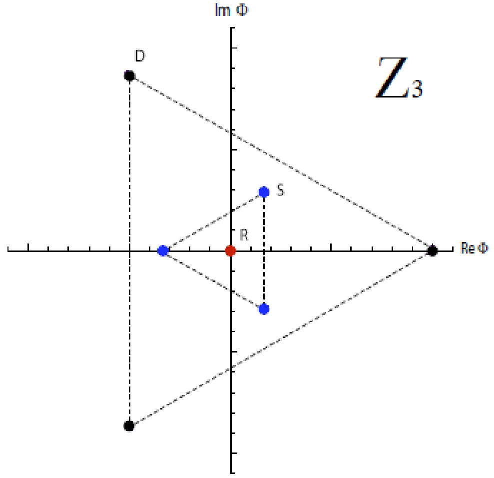
Phase Structure

Lattice data : G. Cossu, M. D'Elia, JHEP 07(2009) 048.
( 4D, two flavor, three color, staggered adjoint fermion )



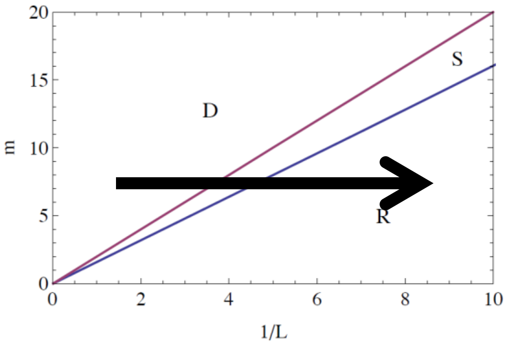

Phase diagram



## Hosotani mechanism

K.K. and T. Misumi, JHEP 05 (2013) 042

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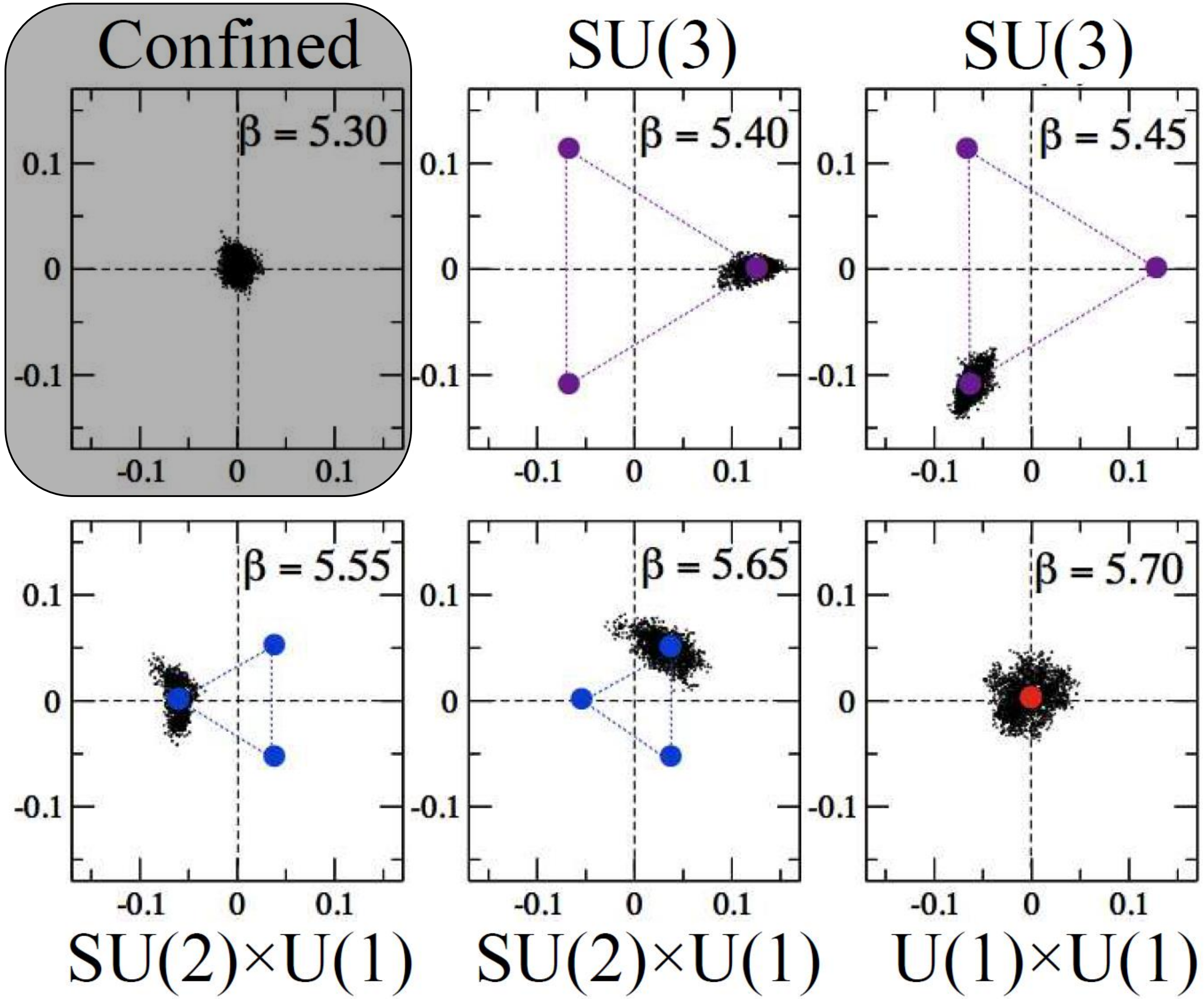
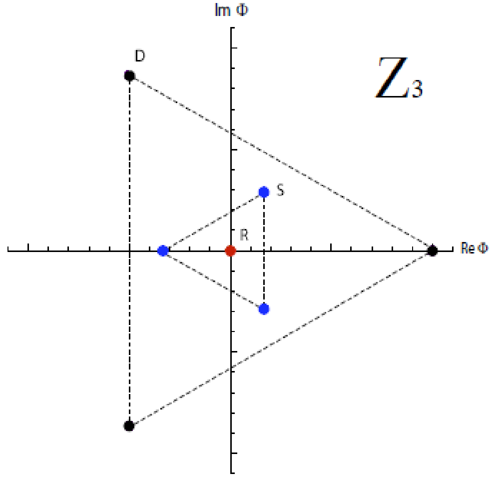


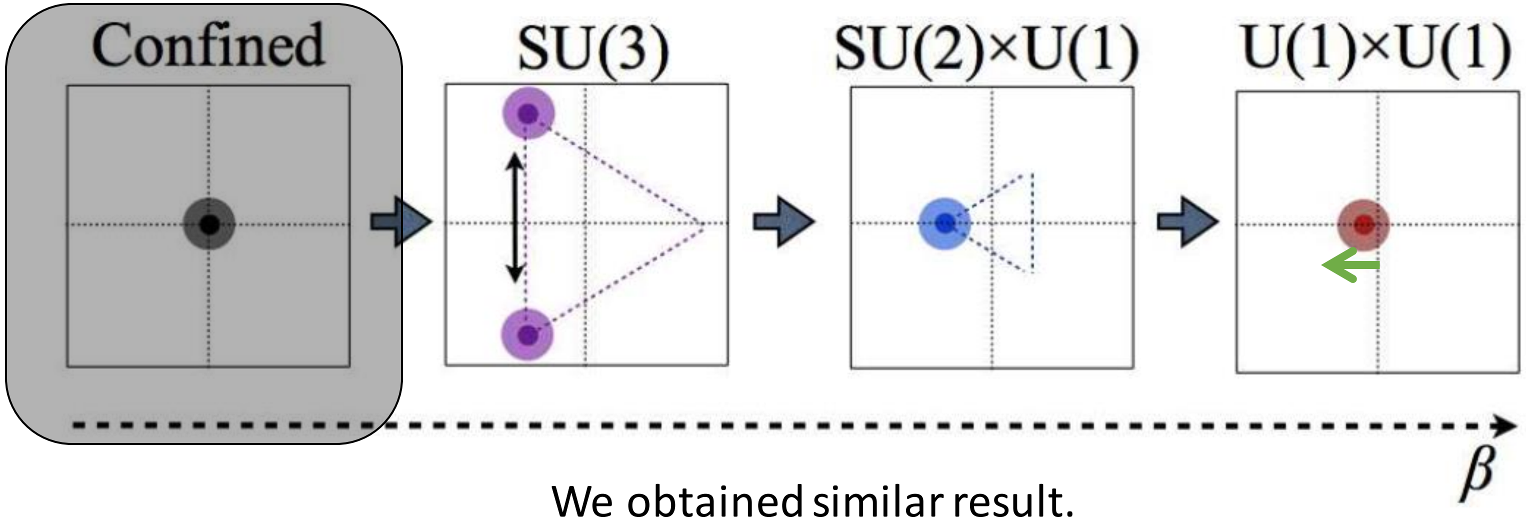

These phases can be understood from Hosotani mechanism!

More detailed lattice results will be shown by
G. Cossu, H. Hatanaka, Y. Hosotani, E. Itou and J. Noaki.

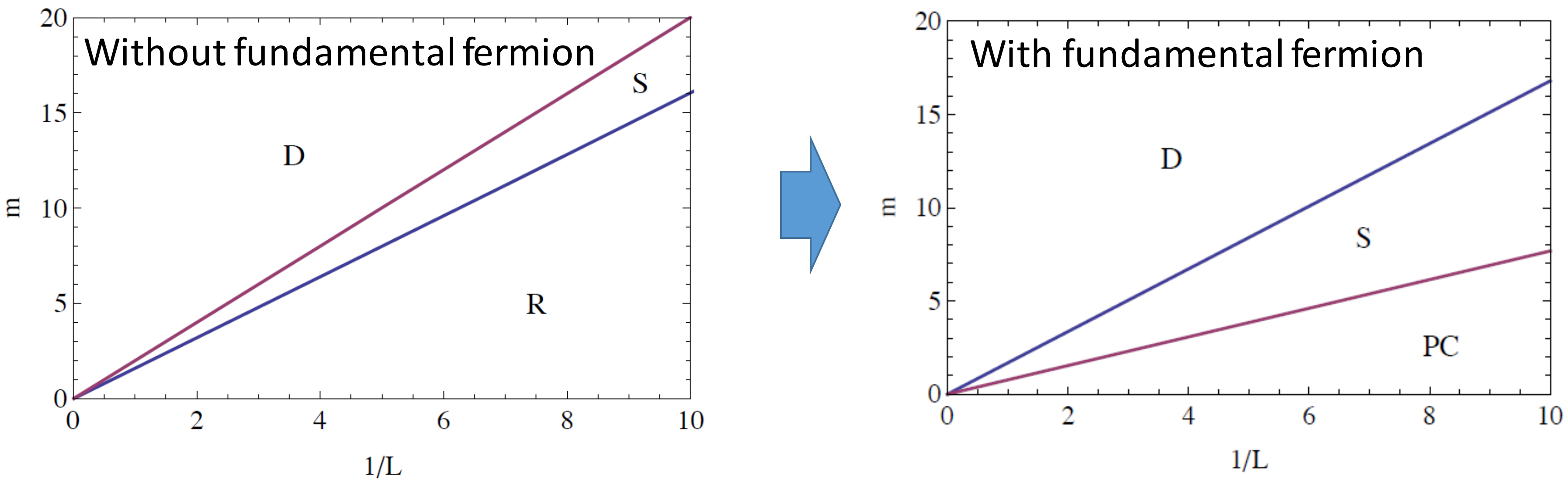
K.K. and T. Misumi, JHEP 05 (2013) 042

+ Fundamental fermion

Fundamental fermion breaks the center symmetry explicitly.




We obtained similar result.



# Chiral symmetry breaking and restoration

K.K. and T. Misumi, JHEP 05 (2013) 042

To describe the chiral symmetry breaking and restoration,
we used the Polyakov-loop extended Nambu—Jona-Lasinio type model.

K. Fukushima, Phys. Lett. B591 (2004) 277.
H. Nishimura and M. Ogilvie, Phys. Rev.D81 (2010) 014018.

There are several discussion about the chiral symmetry breaking.

ex.) M. Unsal, Phys. Rev. Lett. 100 (2008) 032005.

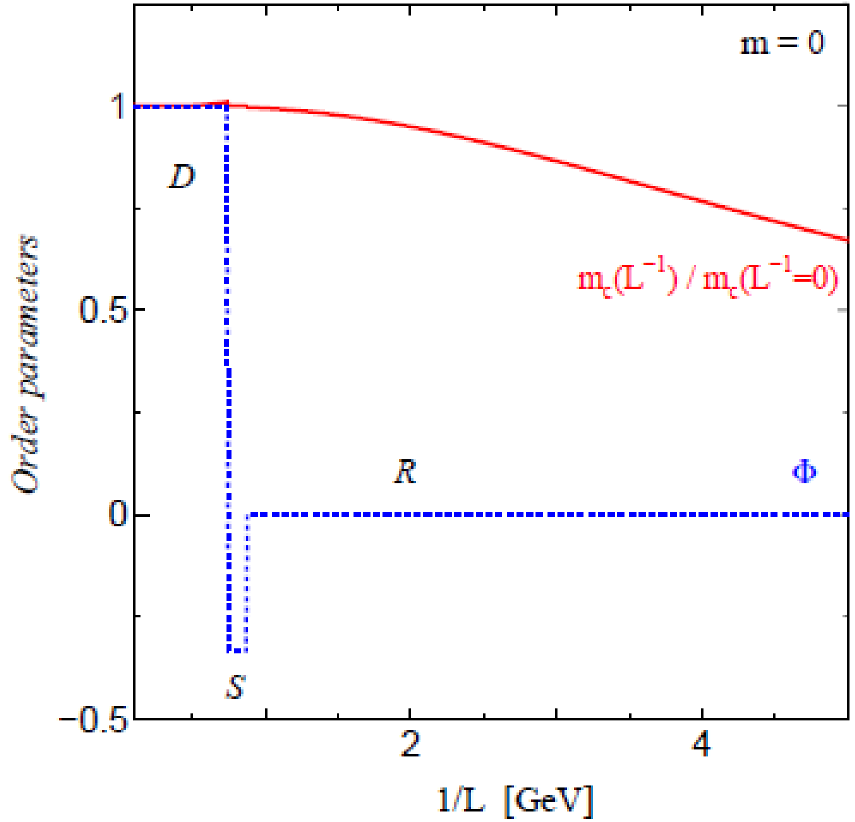
4Fermi interaction:

$$(g_S)_f[(\bar{\psi}_f\psi_f)^2 + (\bar{\psi}_f i\gamma_5\vec{\tau}\psi_f)^2] + (g_S)_a[(\bar{\psi}_a\psi_a)^2 + (\bar{\psi}_a i\gamma_5\vec{\tau}\psi_a)^2]$$
$$+ (g_S)_{fa}[\{(\bar{\psi}_f\psi_f)^2 + (\bar{\psi}_f i\gamma_5\vec{\tau}\psi_f)^2\}^2\{(\bar{\psi}_a\psi_a)^2 + (\bar{\psi}_a i\gamma_5\vec{\tau}\psi_a)^2\}^2]$$

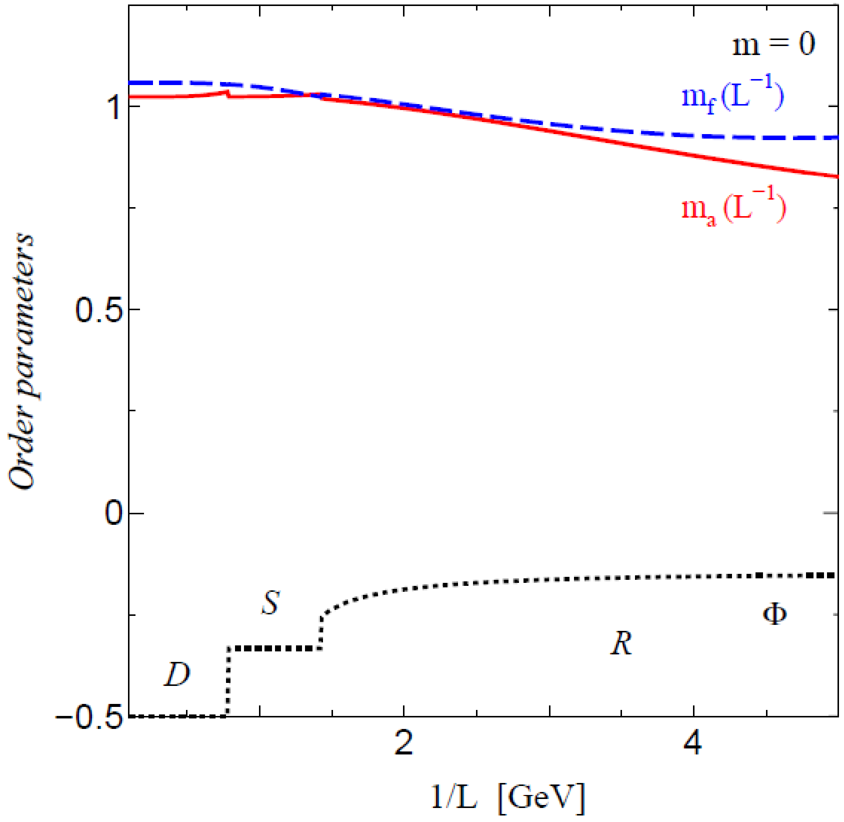
Propagation of non-analyticities

A. Barducci, R. Casalbuoni, G. Pettini, and R. Gatto, Phys. Lett. B 301 (1993) 95.
K. Kashiwa, Y. Sakai, H. Kouno and M. Yahiro, J. Phys. G36 (2009) 105001.

## With adjoint fermion




## With adjoint and fundamental fermion



## Fundamental fermion with flavor twisted boundary condition

H. Kouno, Y. Sakai, T. Makiyama, K. Tokunaga, T. Sasaki, and M. Yahiro, J. Phys. G: Nucl. Part. Phys. **39** (2012) 085010.

We can get the gauge symmetry breaking by using fundamental quark if we consider the **flavor twisted boundary condition**.

H. Kouno, T. Misumi, **K.K.**, T. Makiyama, T. Sasaki, M. Yahiro, Phys. Rev .D 88 (2013) 016002.

Similar one (species-dependent imaginary chemical potential ) has been recently considered in the doubling problem of lattice formalism.

T. Misumi, JHEP 08 (2012) 068.

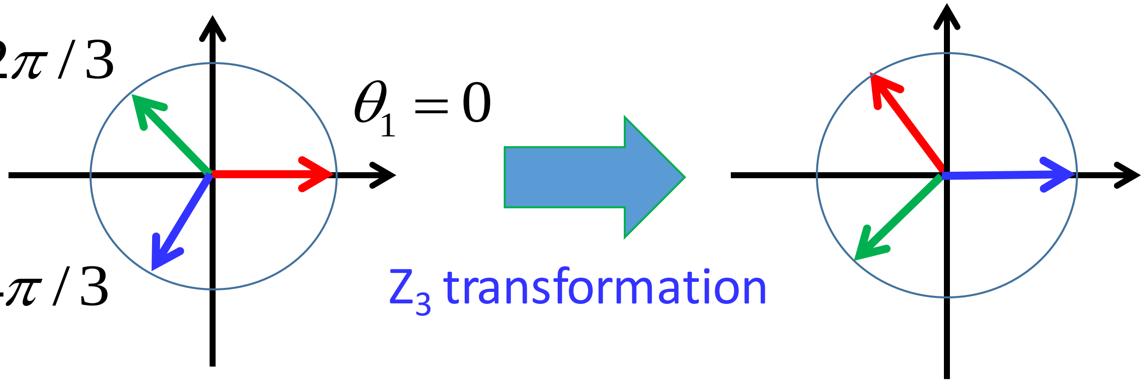
**Flavor twisted boundary condition**

Flavor and color numbers should be same.

$$q_f(x, \beta = 1/T) = -\exp[i\theta_f] q_f(x,0)$$

$$\theta_2 = 2\pi/3$$

$$\theta_3 = 4\pi/3$$




$$\boxed{(q_1, q_2, q_3)_{x,y+L} = (q_1, e^{2\pi i/3} q_2, e^{4\pi i/3} q_3)_{x,y}}$$

cf.) Flavored chemical potential

$\downarrow$ $Z_3$ transformation

$(e^{2\pi i/3} q_1, e^{4\pi i/3} q_2, q_3)_{x,y}$ Relabeling

$Z_3$ center is preserved by use of $Z_3$ of flavor SU(3).

**Fundamental fermion with FTBC**

$$\mathcal{V}_f^{FT} = +\frac{4}{L^4\pi^2} \sum_i^3 \sum_f^3 \sum_{n=1}^{\infty} \frac{\cos[2\pi n q_{if}]}{n^4}$$

This part can be treated as the adjoint trace

$$q_{if} = q_i + (f-1)/3$$

**Adjoint fermion**

$$\mathcal{V}_a = +\frac{4}{L^4\pi^2} \sum_{i,j=1}^{3} \sum_{n=1}^{\infty} \left(1 - \frac{1}{3}\delta_{ij}\right) \frac{\cos[2\pi n q_{ij}]}{n^4}$$

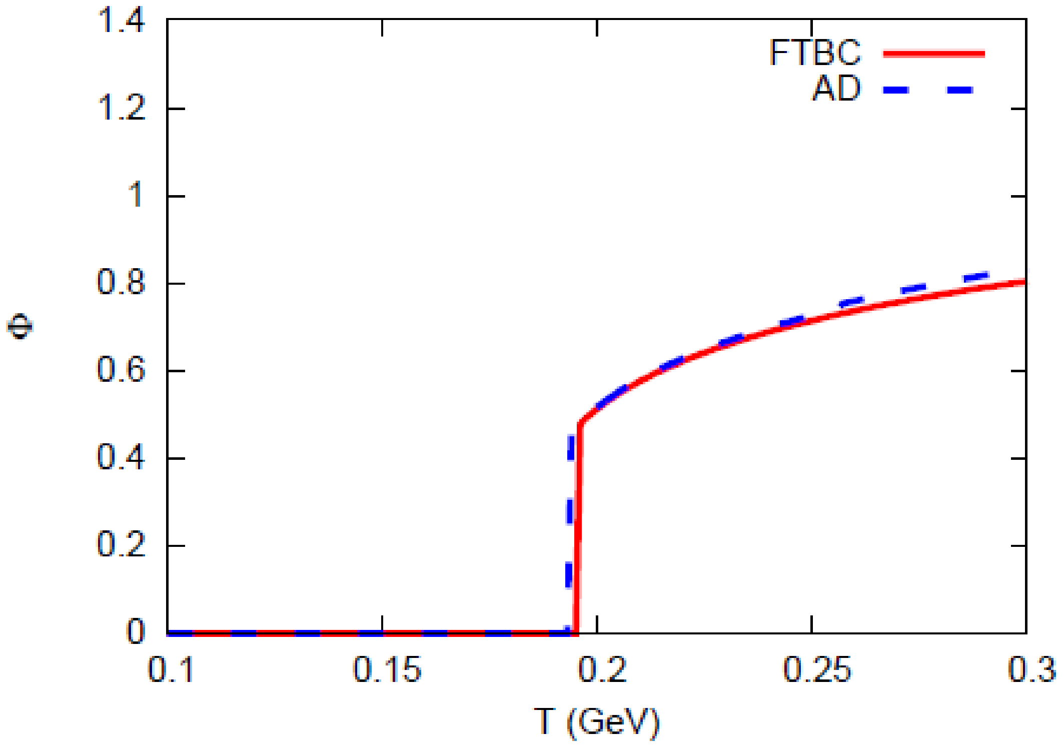
# Fundamental fermion with flavor twisted boundary condition

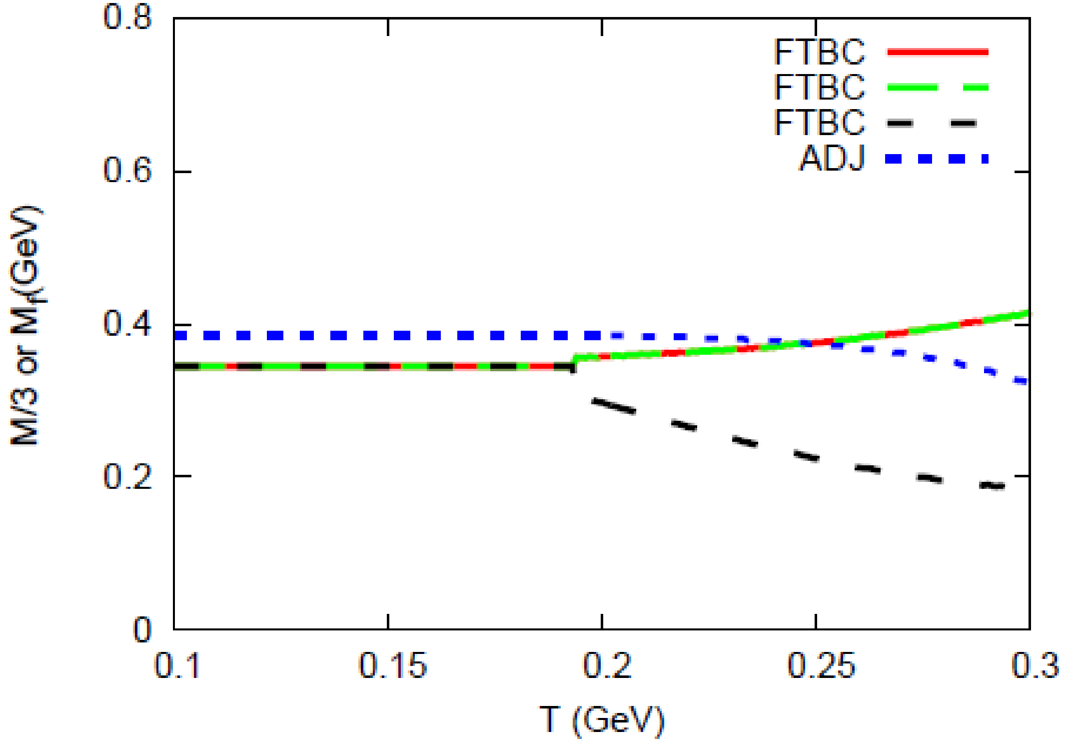
H. Kouno, T. Misumi, **K.K.**, T. Makiyama, T. Sasaki, M. Yahiro,
Phys. Rev .D 88 (2013) 016002.

Polyakov-loop

4D, PNJL model

Constituent quark mass




The adjoint quark and fundamental quark with flavor twisted boundary condition shows similar behavior of the Polyakov-loop.

The constituent quark mass behaves differently because the flavor symmetry breaking is happen in the fundamental fermion with FTBC.

$$\mathcal{V}_f^{FT} = +\frac{4}{L^4\pi^2}\sum_i^3\sum_f^3\sum_{n=1}^{\infty}\frac{\cos[2\pi n q_{if}]}{n^4} \longleftarrow$$

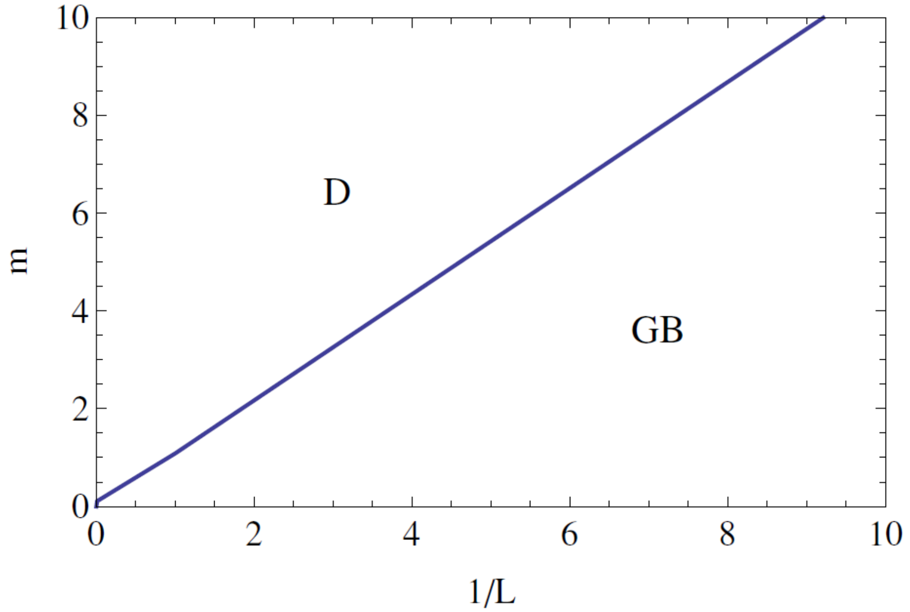
Flavor and color space is strongly entangled in the case.

# Phase diagram and distribution plot of Polyakov-loop

H. Kouno, T. Misumi, **K.K.**, T. Makiyama, T. Sasaki, M. Yahiro,
Phys. Rev .D 88 (2013) 016002.

We get the gauge-symmetry breaking by using fundamental fermion if we consider the **flavor twisted boundary condition**.

Phase diagram

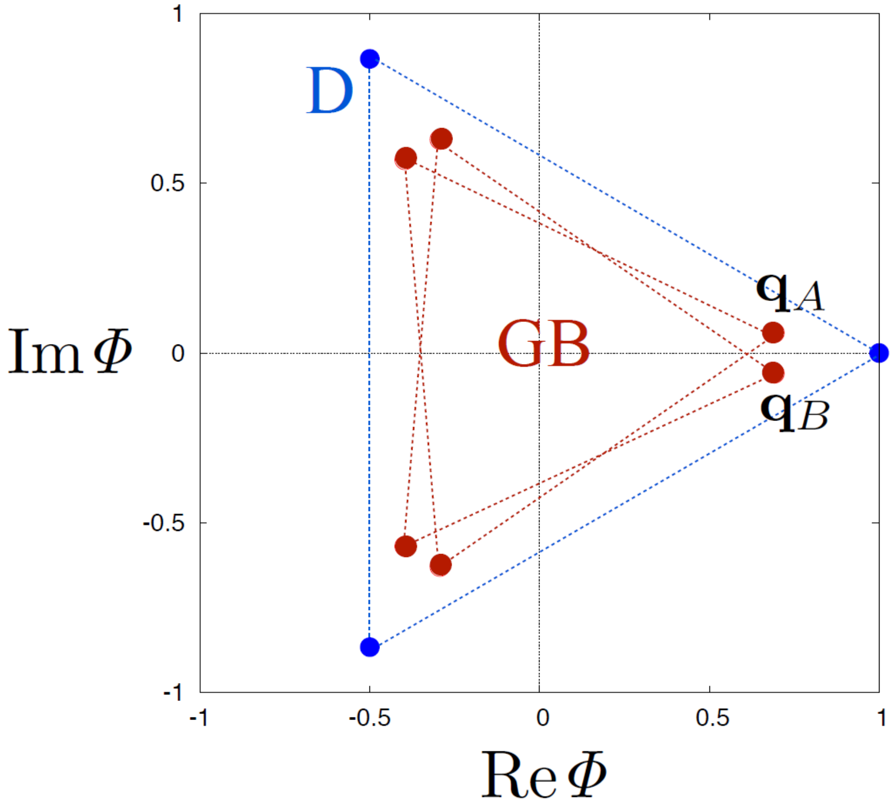



These results are obtained in 4+1dimensional system.

There is the gauge symmetry breaking (SU(2) × U(1)) phase.

$Z_3$ symmetry is not explicitly broken.

Distribution plot of Polyakov-loop




Charge conjugation pairs are appeared in the gauge symmetry broken phase.

# Summary

The **spontaneous gauge symmetry breaking** is discussed by using the adjoint and fundamental fermions.

Effects of the boundary condition are investigated. (periodic, anti-periodic and **flavor twisted boundary conditions**)

The boundary angle can be treated as the **imaginary chemical potential**,
and thus we can use some knowledge obtained in the investigation of QCD phase diagram.

By using the perturbative one-loop effective potential, we can explain the lattice QCD data from **Hosotani mechanism**.

More detailed lattice results will be shown. (G. Cossu, H. Hatanaka, Y. Hosotani, E. Itou and J. Noaki)

By using the flavor twisted boundary condition, we can construct the **center symmetric effective potential**,
and it shows the spontaneous gauge symmetry breaking.

In this theory, there is clear center symmetry breaking and thus,
we can clearly investigate the correlation between the chiral and deconfinement transition of QCD.

The lattice simulation of this theory is very interesting!

Actual LQCD results will be shown soon.

(E. Itou, T. Iritani, T. Misumi …)