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Finite-temperature holographic QCD in the Veneziano limit

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Outline

1. Introduction and motivation
2. Holographic V-QCD models at finite temperature

   [MJ, Kiritsis arXiv:1112.1261]

   [Alho, MJ, Kajantie, Kiritsis, Tuominen arXiv:1210.4516]
3. Finite (temperature and) chemical potential

   [Alho, MJ, Kajantie, Kiritsis, Rosen, Tuominen, work in progress]

Motivation

QCD: $SU(N_c)$ gauge theory with N_f quark flavors (fundamental)

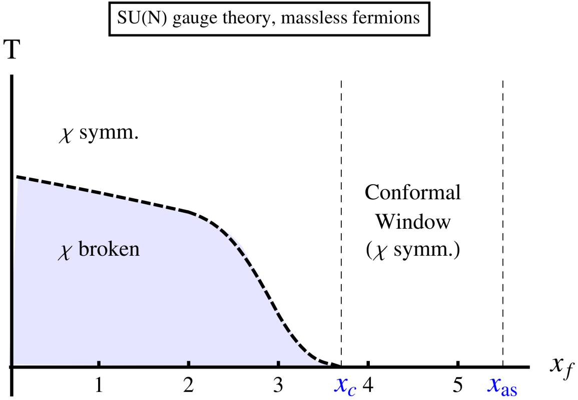
- Often useful: "quenched" or "probe" approximation, $N_f \ll N_c$
- Here Veneziano limit: large N_f, N_c with $x = N_f/N_c$ fixed $\Rightarrow$ backreaction

Veneziano limit, backreaction $\Rightarrow$ better modeling of QCD?
Important new features, mostly not captured by probe limit:

- QCD at finite T, μ, as a function of x
- Already at zero T, μ, nontrivial structure varying $x = N_f/N_c$

Particularly interesting regime: large μ and/or x, where lattice computations have issues

Motivation: expected phase diagram



- Conformal transition at $x = N_f/N_c \simeq 4$
- Walking/quasi-conformal regime + Miransky scaling below the transition

Holographic V-QCD: the fusion

The fusion:

1. IHQCD: bottom-up model for pure glue by using dilaton gravity
   [Gursoy, Kiritsis, Nitti; Gubser, Nellore]
2. Adding flavor and chiral symmetry breaking via tachyon brane actions
   [Klebanov,Maldacena; Bigazzi,Casero,Cotrone,Iatrakis,Kiritsis,Paredes]

Consider 1 + 2 in the Veneziano limit with full backreaction
$\Rightarrow$ V-QCD models

[MJ, Kiritsis arXiv:1112.1261]

Defining V-QCD

Degrees of freedom are two scalar fields:

- The tachyon $\tau \leftrightarrow \bar{q}q$, and the dilaton $\lambda \leftrightarrow \mathrm{Tr}F^2$
- λ is identified as the 't Hooft coupling $g^2 N_c$

$$\mathcal{S}_{\mathrm{V-QCD}} = N_c^2 M^3 \int d^5x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) \right]$$
$$- N_f N_c M^3 \int d^5x V_f(\lambda, \tau) \sqrt{-\det(g_{ab} + \kappa(\lambda) \partial_a \tau \partial_b \tau)}$$

$$V_f(\lambda, \tau) = V_{f0}(\lambda) e^{-a(\lambda)\tau^2} \; ; \quad ds^2 = e^{2A(r)} \left(\frac{dr^2}{f(r)} - f(r) dt^2 + d\mathbf{x}^2 \right)$$

- Need to choose V_g, V_{f0}, a, and κ ...
  - Match with QCD behavior at qualitative level
- The simplest and most reasonable choices do the job!

Various solutions

Solve EoMs, with fifth coordinate $\leftrightarrow$ energy scale

- UV boundary: contact to field theory
- IR structure: several solutions, leading to phase structure

Two classes of IR geometries:

1. Black hole solutions
   - $f'(r_h) = -4\pi T$; $s = 4\pi M^3 N_c^2 e^{3A(r_h)}$
2. Thermal gas solutions ($f \equiv 1$)
   - Any T, zero s

Two types of tachyon behavior
(quark mass and condensate from UV boundary conditions):

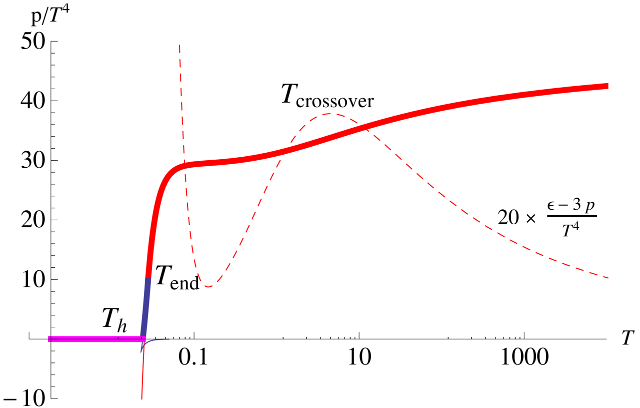
1. Vanishing tachyon – chirally symmetric
2. Nontrivial tachyon – chirally broken

$\Rightarrow$ four types of background solutions

Calculate free energy or pressure in each case, determine the dominant solution

Thermodynamics

Pressure and interaction measure

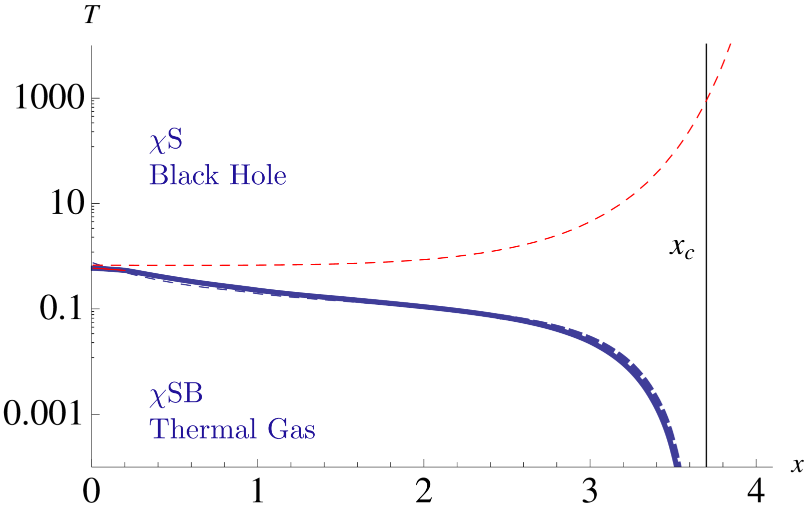


Low T: thermal gas $\longrightarrow$ High T: black hole

Zero pressure at low T due to missing pion loops

Phase diagram: example

Phases on the (x, T)-plane



Turning on finite chemical potential

Work in progress!

Standard method: add a gauge field A_μ dual to $\bar{q}\gamma^\mu q$

$$\mathcal{S}_{\mathrm{V-QCD}} = N_c^2 M^3 \int d^5x \sqrt{g} \left[R - \frac{4}{3}\frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) \right]$$
$$- N_f N_c M^3 \int d^5x V_f(\lambda,\tau)$$
$$\times \sqrt{-\det(g_{ab} + \kappa(\lambda)\partial_a\tau\partial_b\tau + w(\lambda)F_{ab})}$$

$$A_0 = \mu - nr^2 + \cdots$$

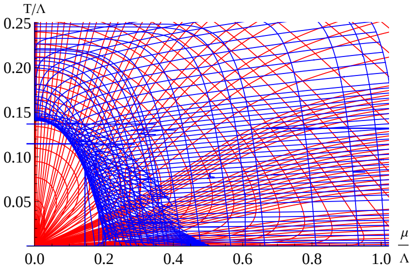
A_0 can be integrated out $\Rightarrow$ one integration constant, which can be mapped to μ

Computation of pressure

Three phases (plots by Timo Alho)

Phases mapped to (μ, T)-plane

- Tachyonic Thermal gas (ChSB), all μ, T (not shown)
- Tachyonless BH (red)
- Tachyonic BH (blue)

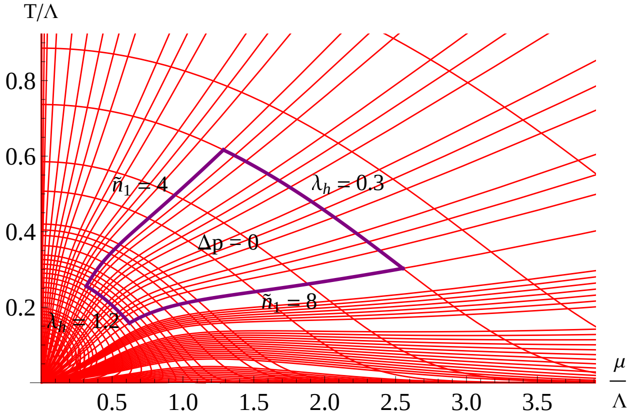


Integrate

$$dp = s\, dT + n\, d\mu$$

along the lines shown

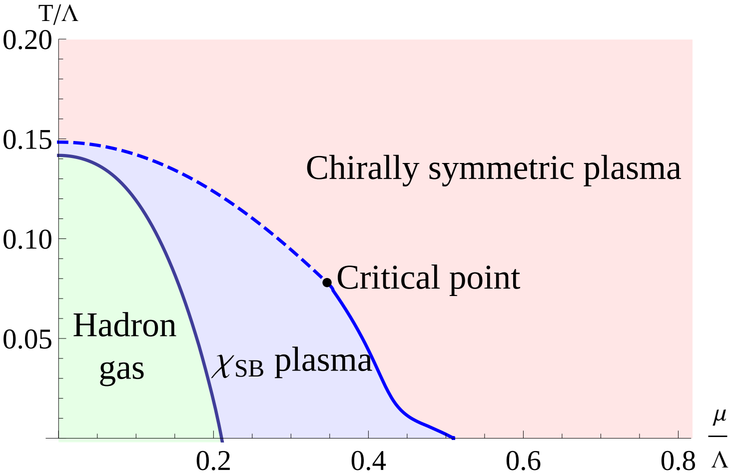
Path independence of pressure



- Path independence verified numerically
- Highly nontrivial check of the consistency of the model and the numerics

Phase diagram (example)

First attempt: $x = N_f/N_c = 1$, Veneziano limit, zero quark mass



Summary

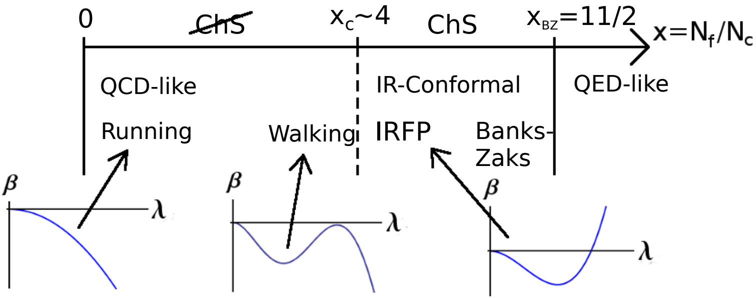
- V-QCD models meet expectations from QCD at qualitative level
- Analysis of structure at finite chemical potential in progress – first results obtained
- Future work: quantitative fit to QCD (lattice + experiments) – towards a more realistic model

Extra slides

QCD phases in the Veneziano limit

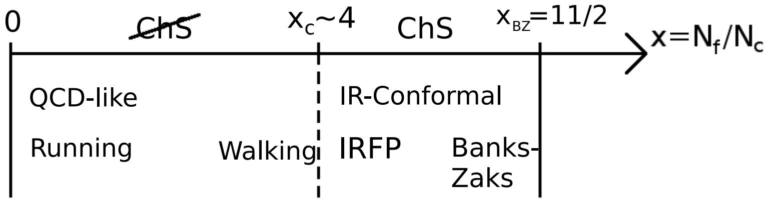
Expected structure at zero T, μ, and quark mass; finite $x = N_f/N_c$

- Phases:
  - $0 < x < x_c$: QCD-like IR, chiral symmetry broken
  - $x_c \leq x < 11/2$: Conformal window, chirally symmetric
- Conformal transition at $x = x_c$
- RG flow of the coupling: running, walking, or fixed point



Phase diagram

Fixing the potentials reasonably, at zero quark mass, after some analysis:



- Meets standard expectations from QCD!
- Conformal transition at $x \simeq 4$ [Kaplan,Son,Stephanov;Kutasov,Lin,Parnachev]

Matching to QCD

In the UV ($\lambda \to 0$):

- UV expansions of potentials matched with perturbative QCD beta functions $\Rightarrow$

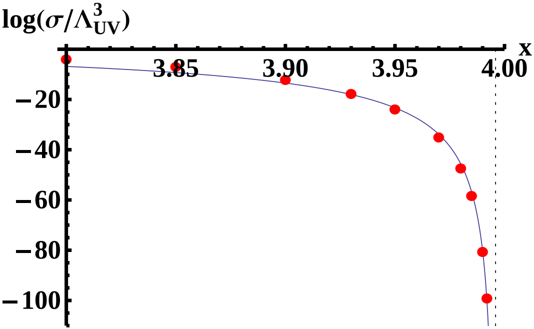
$$\lambda(r) \simeq -\frac{\beta_0}{\log r} \qquad \tau(r) \simeq m(-\log r)^{-\gamma_0/\beta_0}\, r + \sigma(-\log r)^{\gamma_0/\beta_0}\, r^3$$

  with $r \sim 1/\mu \to 0$

In the IR ($\lambda \to \infty$):

- $V_g(\lambda)$ chosen as for Yang-Mills, so that a "good" IR singularity exists
- $V_{f0}(\lambda)$, $a(\lambda)$, and $\kappa(\lambda)$ chosen to produce tachyon divergence: several possibilities ($\to$ Potentials I and II)
- Extra constraints from the asymptotics of the meson spectra

Other important features



$$\langle \bar{q} q \rangle \sim \sigma \sim \exp\left(-\frac{\kappa}{\sqrt{x_c - x}}\right)$$

1. Miransky/BKT scaling as $x \to x_c$ from below
   - E.g., The chiral condensate $\langle \bar{q} q \rangle \propto \sigma$
     (From tachyon UV $\tau(r) \sim m_q(\log r)\, r + \sigma(\log r)\, r^3$)
2. Unstable Efimov vacua observed for $x < x_c$
3. Turning on the quark mass possible

Finite temperature – definitions

Lagrangian as before

$$\mathcal{S}_{\mathrm{V-QCD}} = N_c^2 M^3 \int d^5x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) \right]$$
$$- N_f N_c M^3 \int d^5x \, V_f(\lambda, \tau) \sqrt{-\det(g_{ab} + \kappa(\lambda)\partial_a\tau\partial_b\tau)}$$

A more general metric, A and f solved from EoMs

$$ds^2 = e^{2A(r)} \left(\frac{dr^2}{f(r)} - f(r)dt^2 + d\mathbf{x}^2 \right)$$

Black hole thermodynamics:

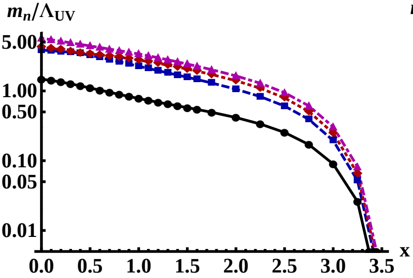
$$f(r) = 4\pi T (r_h - r) + \mathcal{O}\left((r - r_h)^2\right) \; ; \qquad s = 4\pi M^3 N_c^2 e^{3A(r_h)}$$

Also: Thermal gas solutions ($f \equiv 1$, $\sim$ zero T solutions)

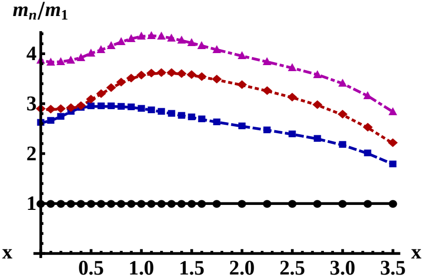
Scalar singlet masses

Scalar singlet spectrum (PotII):

In log scale



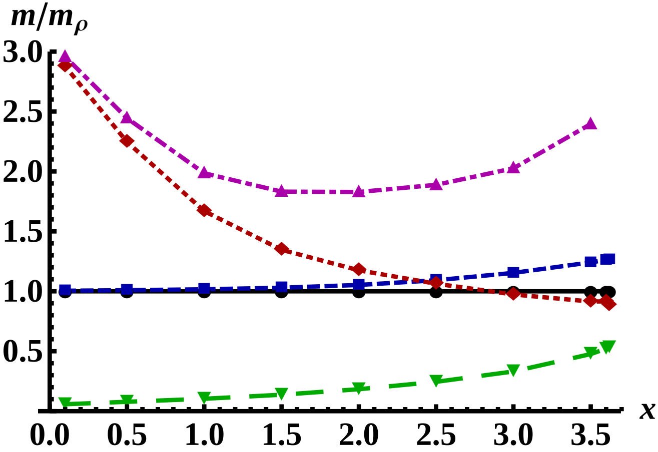
Normalized to the lowest state



No light dilaton?

Meson mass ratios

Mass ratios (PotII): Lowest states normalized to ρ



All ratios tend to constants as $x \to x_c$: indeed no dilaton

Matching to QCD

Similar strategy as in IHQCD

Matching in the UV ($\lambda \to 0$):

- Take analytic potentials at $\lambda = 0$
  $\Rightarrow$ RG flow consistent with QCD (when $A \leftrightarrow \log \mu$)
- Require correct (naive) operator dimensions in the deep UV
- Match expansions of potentials with perturbative QCD beta functions
  - $V_g(\lambda)$ with (two-loop) Yang-Mills beta function
  - $V_g(\lambda) - xV_{f0}(\lambda)$ with QCD beta function
  - $a(\lambda)/\kappa(\lambda)$ with the anomalous dimension of the quark mass/chiral condensate ($\Rightarrow$ properly running quark mass!)
- After this, a single undetermined parameter in the UV: W_0

$$V_{f0}(\lambda) = W_0 + W_1\lambda + \mathcal{O}(\lambda^2)$$

In the IR ($\lambda \to \infty$), there must be a solution where the tachyon action $\propto e^{-a(\lambda)\tau^2} \to 0$

- $V_g(\lambda)$ chosen as for Yang-Mills, so that a "good" IR singularity exists
- $V_{f0}(\lambda)$, $a(\lambda)$, and $\kappa(\lambda)$ chosen to produce tachyon divergence: several possibilities ($\to$ Potentials I and II)
- Extra constraints from the asymptotics of the meson spectra

Working potentials often string-inspired power-laws, multiplied by logarithmic corrections (!)

Background analysis: zero temperature

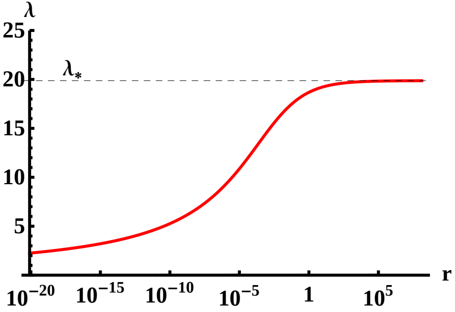
Analysis of the backgrounds (r-dependent solutions of EoMs) at zero temperature

- Expect two kinds of solutions, with
  1. Nontrivial tachyon profile (chirally broken)
  2. Identically vanishing tachyon (chirally symmetric)
- Identify the dominant vacua
- Fully backreacted system $\Rightarrow$ rich dynamics, complicated numerical analysis . . .

Backgrounds at zero quark mass

Sketch of behavior in the conformal window ($x > x_c$):

- Tachyon vanishes (no ChSB)
- Similar to IHQCD, different potential
  $\Rightarrow$ IR fixed point
- Dilaton flows between UV/IR fixed points

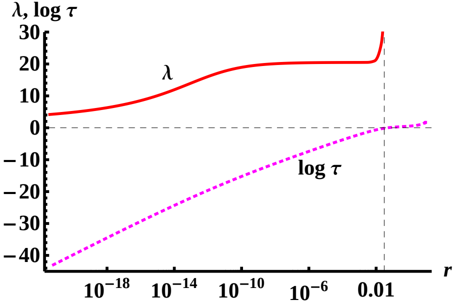


Here UV: $r \to 0$, IR: $r \to \infty$

As x goes below x_c, this solution becomes unstable (tachyon BF bound)

Right below the conformal window ($x < x_c$; $|x - x_c| \ll 1$)

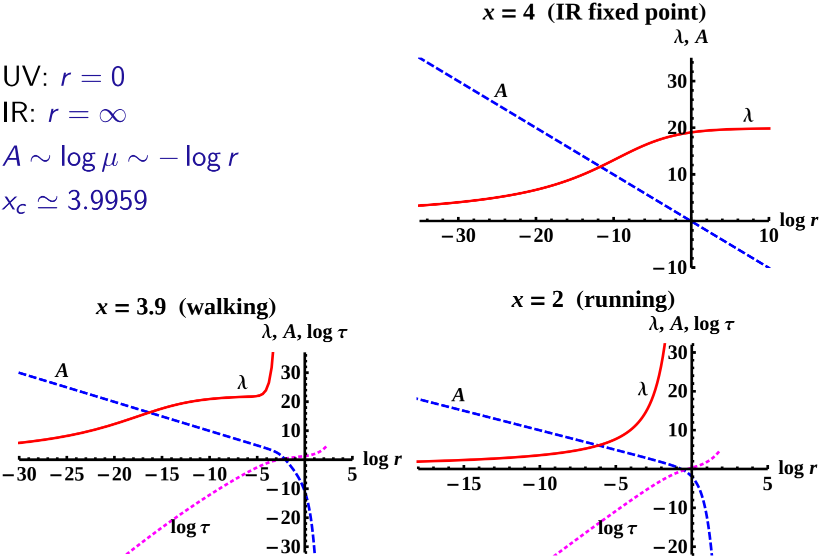
- Dilaton flows very close to the IR fixed point
- "Small" nonzero tachyon induces an IR singularity



Result: "walking"

Actual solutions

UV: $r = 0$
IR: $r = \infty$
$A \sim \log \mu \sim -\log r$
$x_c \simeq 3.9959$



The BF bound and x_c

At an fixed point

$$\tau(r) \sim C_1 r^{\Delta} + C_2 r^{4-\Delta}$$

with

$$-m^2\ell^2 = \Delta(4-\Delta)$$

Requiring real Δ gives the Breitenlohner-Freedman bound for the tachyon (Starinets' lectures)

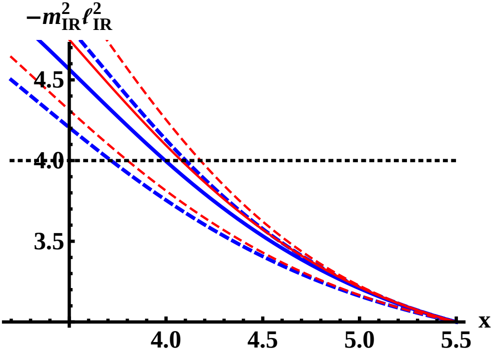
$$-m^2\ell^2 = \Delta(4-\Delta) \leq 4$$

- Saturated for $\Delta = 2$, then $\tau(r) \sim C_1 r^2 + C_2 r^2 \log r$
- Violation of BF bound $\Rightarrow$ instability

Analysis of this instability of the tachyon $\Rightarrow x_c$

Dependence on the UV parameter W_0 and IR choices for the potentials

Resulting variation of the edge of conformal window $x_c = 3.7 \ldots 4.2$



Agrees with most of the other estimates

Potentials I

$$
\begin{aligned}
V_g(\lambda) &= 12 + \frac{44}{9\pi^2}\lambda + \frac{4619}{3888\pi^4}\frac{\lambda^2}{(1+\lambda/(8\pi^2))^{2/3}}\sqrt{1+\log(1+\lambda/(8\pi^2))} \\
V_f(\lambda,\tau) &= V_{f0}(\lambda)e^{-a(\lambda)\tau^2} \\
V_{f0}(\lambda) &= \frac{12}{11} + \frac{4(33-2x)}{99\pi^2}\lambda + \frac{23473-2726x+92x^2}{42768\pi^4}\lambda^2 \\
a(\lambda) &= \frac{3}{22}(11-x) \\
\kappa(\lambda) &= \frac{1}{\left(1+\frac{115-16x}{288\pi^2}\lambda\right)^{4/3}}
\end{aligned}
$$

In this case the tachyon diverges exponentially:

$$
\tau(r) \sim \tau_0 \exp\left[\frac{81\ 3^{5/6}(115-16x)^{4/3}(11-x)}{812944\ 2^{1/6}}\frac{r}{R}\right]
$$

Potentials II

$$
\begin{aligned}
V_g(\lambda) &= 12 + \frac{44}{9\pi^2}\lambda + \frac{4619}{3888\pi^4}\frac{\lambda^2}{(1+\lambda/(8\pi^2))^{2/3}}\sqrt{1+\log(1+\lambda/(8\pi^2))} \\
V_f(\lambda,\tau) &= V_{f0}(\lambda)e^{-a(\lambda)\tau^2} \\
V_{f0}(\lambda) &= \frac{12}{11} + \frac{4(33-2x)}{99\pi^2}\lambda + \frac{23473-2726x+92x^2}{42768\pi^4}\lambda^2 \\
a(\lambda) &= \frac{3}{22}(11-x)\frac{1+\frac{115-16x}{216\pi^2}\lambda+\lambda^2/(8\pi^2)^2}{(1+\lambda/(8\pi^2))^{4/3}} \\
\kappa(\lambda) &= \frac{1}{(1+\lambda/(8\pi^2))^{4/3}}
\end{aligned}
$$

In this case the tachyon diverges as

$$
\tau(r) \sim \frac{27\ 2^{3/4}3^{1/4}}{\sqrt{4619}}\sqrt{\frac{r-r_1}{R}}
$$

Effective potential

For solutions with $\tau = \tau_* = \text{const}$

$$\mathcal{S} = M^3 N_c^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) - x V_f(\lambda, \tau_*) \right]$$

IHQCD with an effective potential

$$V_{\text{eff}}(\lambda) = V_g(\lambda) - x V_f(\lambda, \tau_*) = V_g(\lambda) - x V_{f0}(\lambda) \exp(-a(\lambda)\tau_*^2)$$

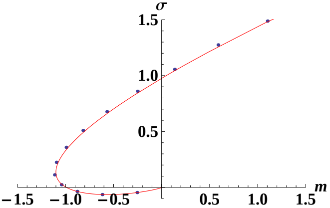
Minimizing for τ_* we obtain $\tau_* = 0$ and $\tau_* = \infty$

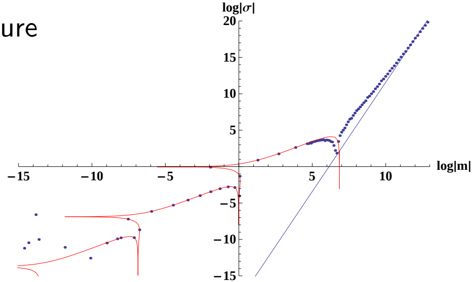
- $\tau_* = 0$: $V_{\text{eff}}(\lambda) = V_g(\lambda) - x V_{f0}(\lambda)$;
  fixed point with $V'_{\text{eff}}(\lambda_*) = 0$
- $\tau_* \to \infty$: $V_{\text{eff}}(\lambda) = V_g(\lambda)$ (like YM, no fixed points)

Efimov spiral

Ongoing work: the dependence $\sigma(m)$ of the chiral condensate on the quark mass

- For $x < x_c$ spiral structure



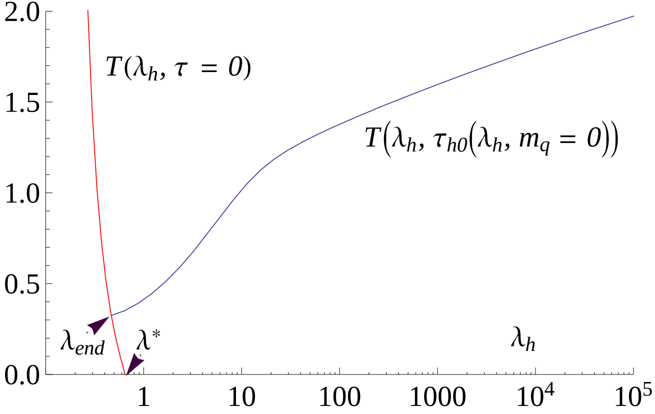


- Dots: numerical data
- Continuous line: (semi-)analytic prediction

Allows to study the effect of double-trace deformations

Black hole branches

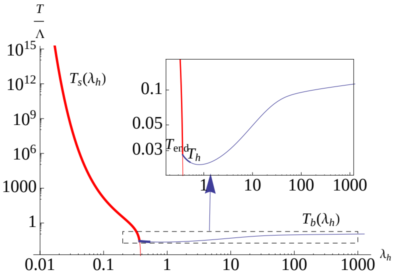
Example: PotII at $x = 3$, $W_0 = 12/11$



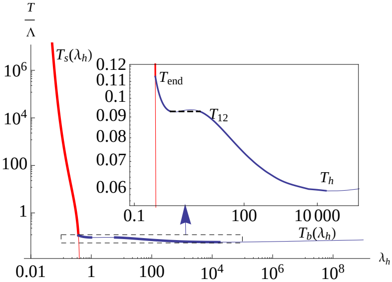
Simple phase structure: 1st order transition at $T = T_h$ from thermal gas to (chirally symmetric) BH

More complicated cases:

PotII at $x = 3$, W_0 SB



PotI at $x = 3.5$, $W_0 = 12/11$

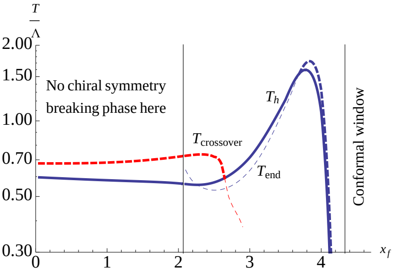


- Left: chiral symmetry restored at 2nd order transition with $T = T_{\text{end}} > T_h$
- Right: Additional first order transition between BH phases with broken chiral symmetry

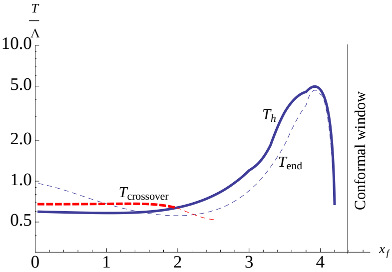
Also other cases ...

Phase diagrams on the (x, T)-plane

PotI$_*$ W_0 SB

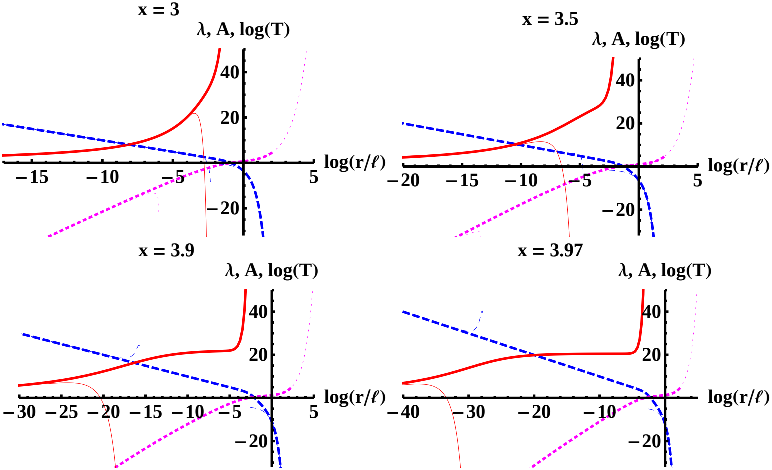


PotII$_*$ W_0 SB

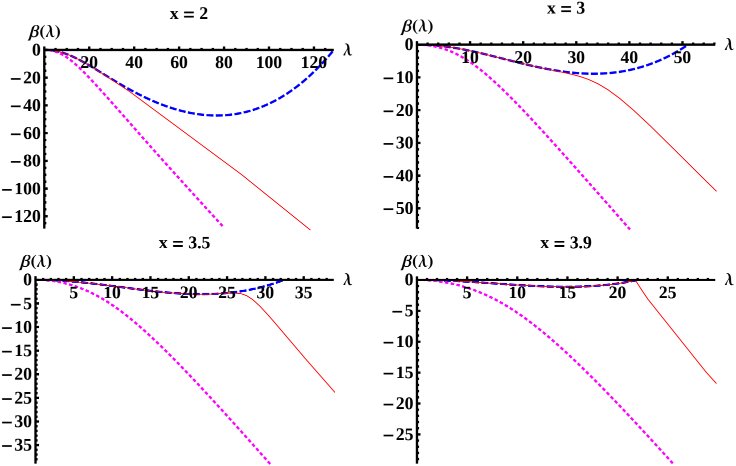


Backgrounds in the walking region

Backgrounds with zero quark mass, $x < x_c \simeq 3.9959$ (λ, A, τ)



Beta functions along the RG flow (evaluated on the background),
zero tachyon, YM $x_c \simeq 3.9959$



Holographic beta functions

Generalization of the holographic RG flow of IHQCD

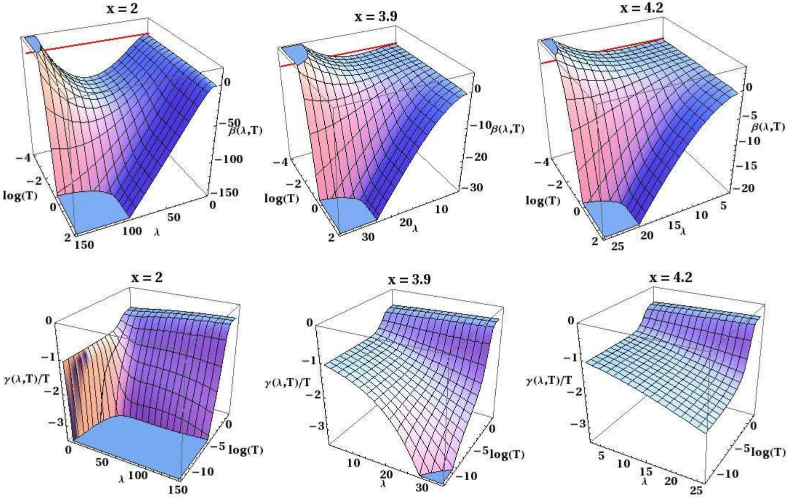
$$\beta(\lambda,\tau) \equiv \frac{d\lambda}{dA}\ ; \qquad \gamma(\lambda,\tau) \equiv \frac{d\tau}{dA}$$

linked to

$$\frac{dg_{\mathrm{QCD}}}{d\log\mu}\ ; \qquad \frac{dm}{d\log\mu}$$

The full equations of motion boil down to two first order partial non-linear differential equations for β and γ

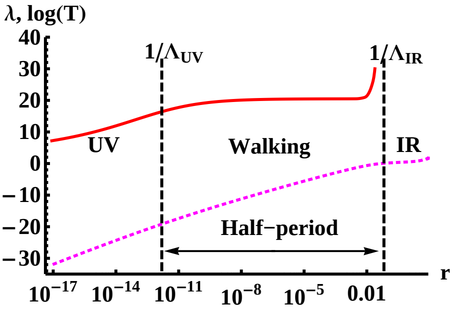
“Good” solutions numerically (unique)



Miransky/BKT scaling

As $x \to x_c$ from below: walking, dominant solution

- BF-bound for the tachyon violated at the IRFP
- x_c fixed by the BF bound: $\Delta = 2$ & $\gamma_* = 1$ at the edge of the conformal window



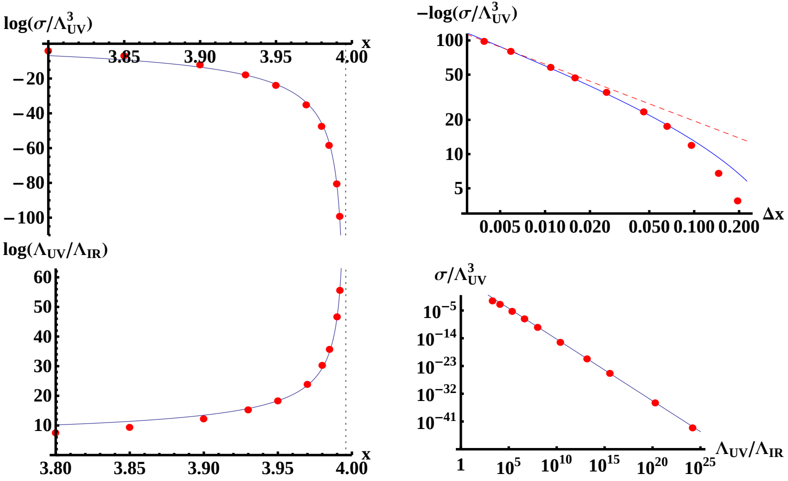
- $\tau(r) \sim r^2 \sin(\kappa\sqrt{x_c - x}\log r + \phi)$ in the walking region
- "0.5 oscillations" $\Rightarrow$ Miransky/BKT scaling, amount of walking $\Lambda_{UV}/\Lambda_{IR} \sim \exp(\pi/(\kappa\sqrt{x_c - x}))$

As $x \to x_c$

with known κ

$$\langle \bar{q} q \rangle \sim \sigma \sim \exp(-2\pi/(\kappa\sqrt{x_c - x}))$$

$$\Lambda_{UV}/\Lambda_{IR} \sim \exp(\pi/(\kappa\sqrt{x_c - x}))$$

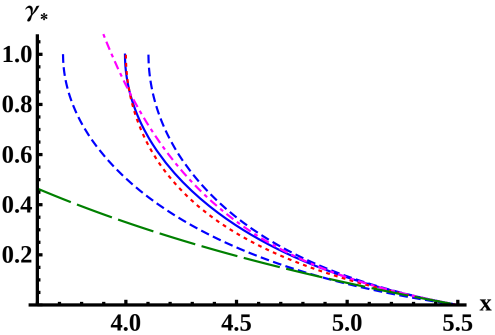


γ_* in the conformal window

Comparison to other guesses

V-QCD (dashed: variation due to W_0)
Dyson-Schwinger
2-loop PQCD
All-orders β

[Pica, Sannino arXiv:1011.3832]



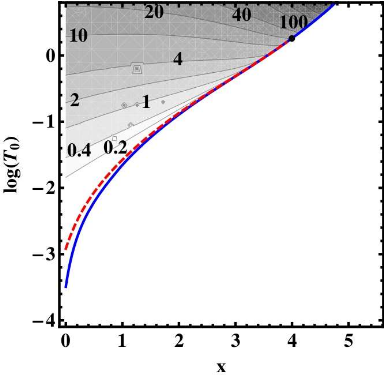
Parameters

Understanding the solutions for generic quark masses requires discussing parameters

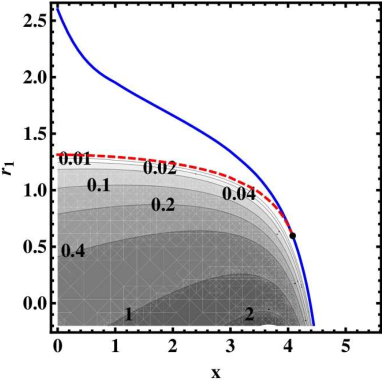
- YM or QCD with massless quarks: no parameters
- QCD with flavor-independent mass m: a single (dimensionless) parameter m/Λ_{QCD}
- In this model, after rescalings, this parameter can be mapped to a parameter (τ_0 or r_1) that controls the diverging tachyon in the IR
- x has become continuous in the Veneziano limit

Map of all solutions

All "good" solutions ($\tau \neq 0$) obtained varying x and τ_0 or r_1
Contouring: quark mass (zero mass is the red contour)

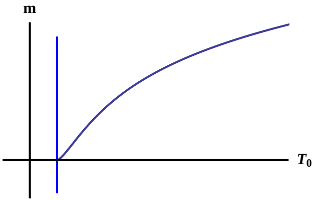


"Potentials I" $\leftrightarrow$ T_0



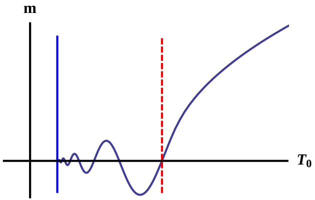
"Potentials II" $\leftrightarrow$ r_1

Mass dependence and Efimov vacua



Conformal window ($x > x_c$)

- For $m = 0$, unique solution with $\tau \equiv 0$
- For $m > 0$, unique "standard" solution with $\tau \neq 0$

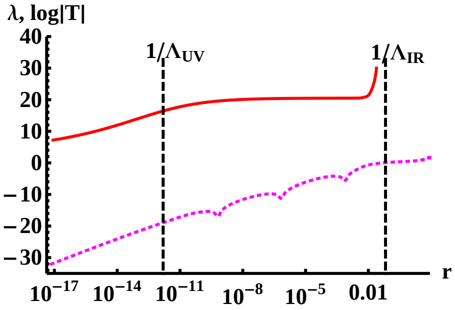


Low $0 < x < x_c$: Efimov vacua

- Unstable solution with $\tau \equiv 0$ and $m = 0$
- "Standard" stable solution, with $\tau \neq 0$, for all $m \geq 0$
- Tower of unstable Efimov vacua (small $|m|$)

Efimov solutions

- Tachyon oscillates over the walking regime
- $\Lambda_{UV}/\Lambda_{IR}$ increased wrt. "standard" solution

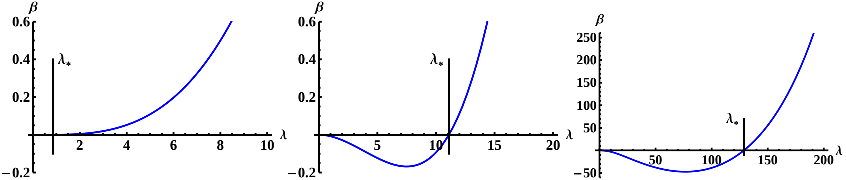


Effective potential: zero tachyon

Start from Banks-Zaks region, $\tau_* = 0$, chiral symmetry conserved $(\tau \leftrightarrow \bar{q}q)$, $V_{\rm eff}(\lambda) = V_g(\lambda) - x V_{f0}(\lambda)$

- $V_{\rm eff}$ defines a β-function as in IHQCD – Fixed point guaranteed in the BZ region, moves to higher λ with decreasing x
- Fixed point λ_* runs to ∞ either at finite $x (< x_c)$ or as $x \to 0$

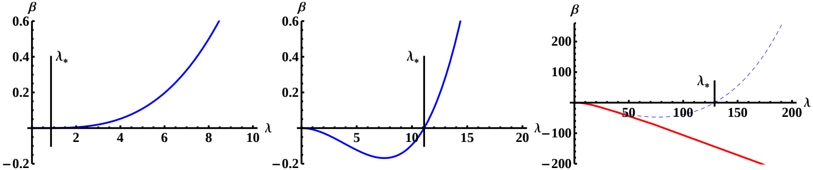
Banks-Zaks	Conformal Window	
$x \to 11/2$	$x > x_c$	$x < x_c$??



Effective potential: what actually happens

Banks-Zaks | Conformal Window

$x \to 11/2$ | $x > x_c$ | $x < x_c$



$\tau \equiv 0$ | $\tau \equiv 0$ | $\tau \neq 0$

- For $x < x_c$ vacuum has nonzero tachyon (checked by calculating free energies)
- The tachyon screens the fixed point
- In the deep IR τ diverges, $V_{\text{eff}} \to V_g \Rightarrow$ dynamics is YM-like

Where is x_c?

How is the edge of the conformal window stabilized?
Tachyon IR mass at $\lambda = \lambda_*$ $\leftrightarrow$ quark mass dimension

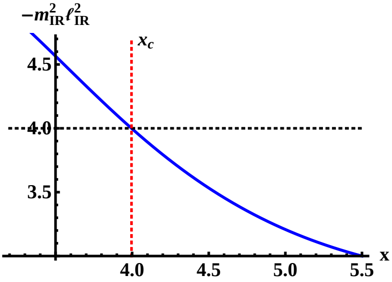
$$-m_{\rm IR}^2 \ell_{\rm IR}^2 = \Delta_{\rm IR}(4 - \Delta_{\rm IR}) = \frac{24 a(\lambda_*)}{\kappa(\lambda_*)(V_g(\lambda_*) - x V_0(\lambda_*))}$$

$$\gamma_* = \Delta_{\rm IR} - 1$$

Breitenlohner-Freedman (BF) bound (horizontal line)

$$-m_{\rm IR}^2 \ell_{\rm IR}^2 = 4 \;\Leftrightarrow\; \gamma_* = 1$$

defines x_c



Why $\gamma_* = 1$ at $x = x_c$?

No time to go into details ... the question boils down to the linearized tachyon solution at the fixed point

- For $\Delta_{\rm IR}(4 - \Delta_{\rm IR}) < 4 \qquad (x > x_c)$:

$$\tau(r) \sim m_q r^{\Delta_{\rm IR}} + \sigma r^{4-\Delta_{\rm IR}}$$

- For $\Delta_{\rm IR}(4 - \Delta_{\rm IR}) > 4 \qquad (x < x_c)$:

$$\tau(r) \sim C r^2 \sin\left[(\mathrm{Im}\Delta_{\rm IR}) \log r + \phi\right]$$

Rough analogy:
Tachyon EoM $\leftrightarrow$ Gap equation in Dyson-Schwinger approach
Similar observations have been made in other holographic frameworks

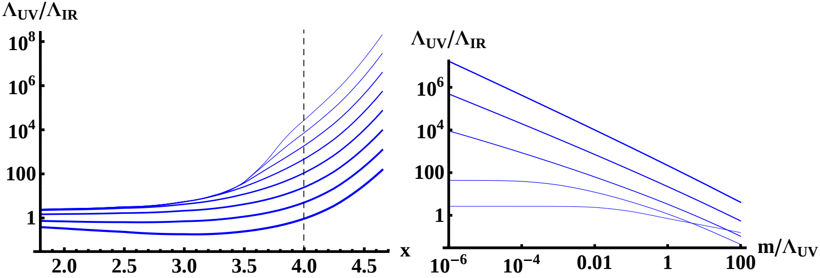
[Kutasov, Lin, Parnachev arXiv:1107.2324, 1201.4123]

Mass dependence

For $m > 0$ the conformal transition disappears

The ratio of typical UV/IR scales $\Lambda_{UV}/\Lambda_{IR}$ varies in a natural way

$m/\Lambda_{UV} = 10^{-6}, 10^{-5}, \ldots, 10 \qquad x = 2, 3.5, 3.9, 4.25, 4.5$



sQCD phases

The case of $\mathcal{N} = 1$ $SU(N_c)$ superQCD with N_f quark multiplets is known and provides an interesting (and more complex) example for the nonsupersymmetric case. From Seiberg we have learned that:

- $x = 0$ the theory has confinement, a mass gap and N_c distinct vacua associated with a spontaneous breaking of the leftover R symmetry Z_{N_c}.
- At $0 < x < 1$, the theory has a runaway ground state.
- At $x = 1$, the theory has a quantum moduli space with no singularity. This reflects confinement with ChSB.
- At $x = 1 + 1/N_c$, the moduli space is classical (and singular). The theory confines, but there is no ChSB.
- At $1 + 2/N_c < x < 3/2$ the theory is in the non-abelian magnetic IR-free phase, with the magnetic gauge group $SU(N_f - N_c)$ IR free.
- At $3/2 < x < 3$, the theory flows to a CFT in the IR. Near $x = 3$ this is the Banks-Zaks region where the original theory has an IR fixed point at weak coupling. Moving to lower values, the coupling of the IR $SU(N_c)$ gauge theory grows. However near $x = 3/2$ the dual magnetic $SU(N_f - N_c)$ is in its Banks-Zaks region, and provides a weakly coupled description of the IR fixed point theory.
- At $x > 3$, the theory is IR free.

Saturating the BF bound (sketch)

Why is the BF bound saturated at the phase transition (massless quarks)??

$$\Delta_{\text{IR}}(4-\Delta_{\text{IR}}) = \frac{24a(\lambda_*)}{\kappa(\lambda_*)(V_g(\lambda_*) - xV_0(\lambda_*))}$$

- For $\Delta_{\text{IR}}(4-\Delta_{\text{IR}}) < 4$:
  $\tau(r) \sim m_q r^{4-\Delta_{\text{IR}}} + \sigma r^{\Delta_{\text{IR}}}$
- For $\Delta_{\text{IR}}(4-\Delta_{\text{IR}}) > 4$:
  $\tau(r) \sim C r^2 \sin\left[(\text{Im}\Delta_{\text{IR}}) \log r + \phi\right]$
- Saturating the BF bound, the tachyon solutions will engtangle $\rightarrow$ required to satisfy boundary conditions
- Nodes in the solution appear trough UV $\rightarrow$ massless solution

Saturating the BF bound (sketch)

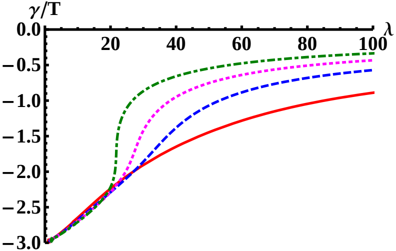
Does the nontrivial (ChSB) massless tachyon solution exist?
Two possibilities:

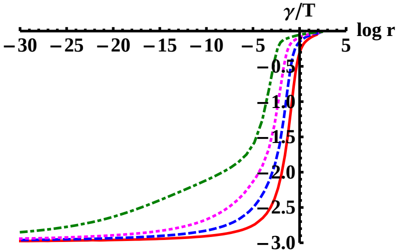
- $x > x_c$: BF bound satisfied at the fixed point $\Rightarrow$ only trivial massless solution ($\tau \equiv 0$, ChS intact, fixed point hit)
- $x < x_c$: BF bound violated at the fixed point $\Rightarrow$ a nontrivial massless solution exist, which drives the system away from the fixed point

Conclusion: phase transition at $x = x_c$
As $x \to x_c$ from below, need to approach the fixed point to satisfy the boundary conditions $\Rightarrow$ nearly conformal, "walking" dynamics

Gamma functions

Massless backgrounds: gamma functions $\frac{\gamma}{\tau} = \frac{d \log \tau}{dA}$





$x = 2, 3, 3.5, 3.9$