

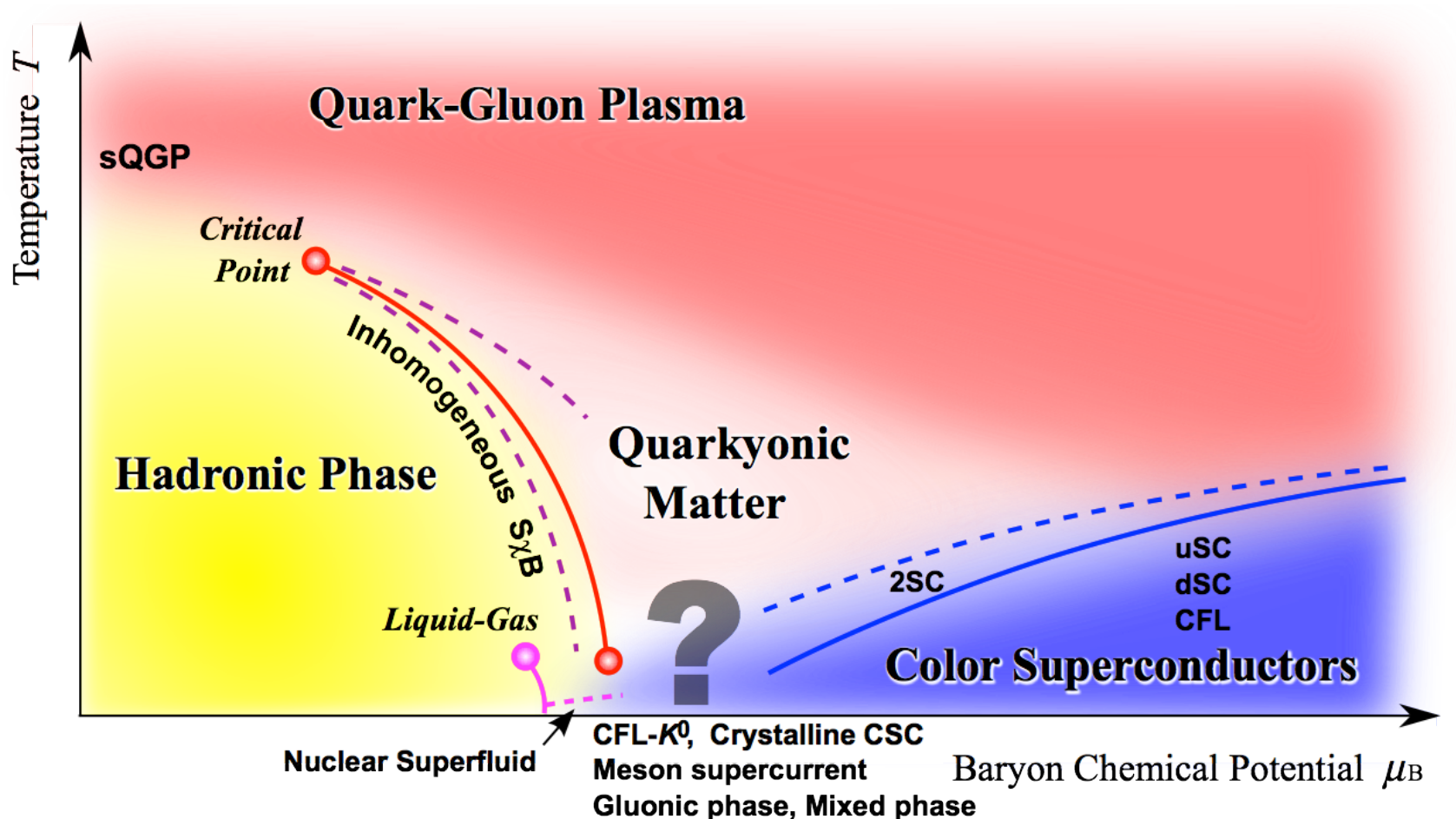
Analytic continuation in two color QCD with clover-improved Wilson fermion

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QCD Phase Diagram

Fukushima and Hatsuda, Rept. Prog. Phys. 74



Sign Problem

$$\langle \mathcal{O} \rangle = \int dU e^{-S[U]} \mathcal{O}[U] \quad U(x_\nu, 4, N_c)$$

very huge multiple integral

MC with importance sampling

$$\text{integrand} \quad e^{-S[U]} \sim \det \Delta(\mu) e^{-S_g[U]}$$

Finite chemical potential μ (Sign Problem)

$$(\det \Delta(\mu))^* = \det \Delta(-\mu^*) \in \mathbb{C}$$

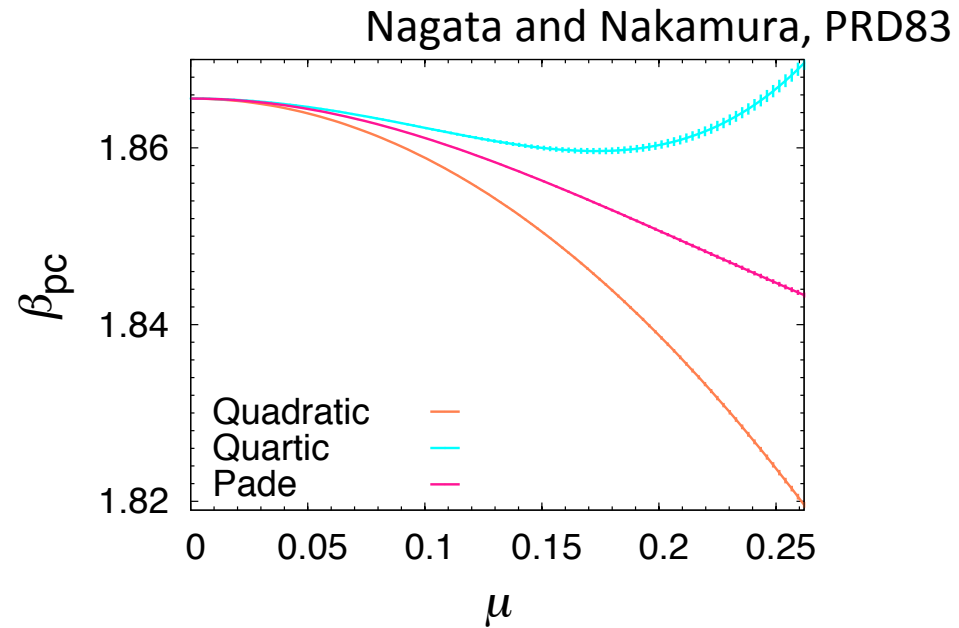
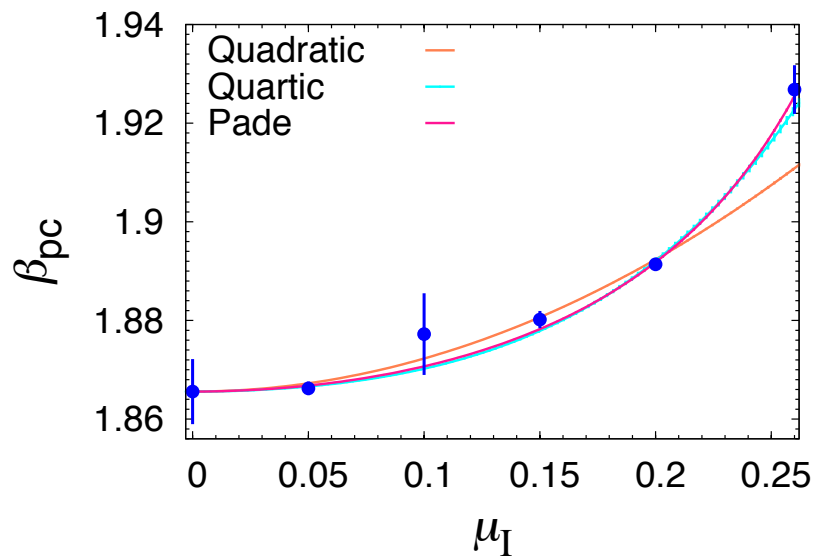
NO sign problem

$$\text{Imaginary } \mu \quad (\det \Delta(i\mu))^* = \det \Delta(i\mu) \in \mathbb{R}$$

Analytic Continuation from imaginary μ

Lattice data can be made analytic continuation from imaginary to real μ .
such as the phase transition line

talked by A. Alexandru



For two color QCD, NO sign problem at both imaginary and real μ .
We can check reliability of the analytic continuation method.

Outline

We investigated two-color QCD phase diagram
at both imaginary and real μ .

We use the RG improved gauge + Clover improved Wilson fermion
on the line of the constant physics with $M_{ps}/M_v=0.80$ in $8^3 \times 4$ size.

We calculated
the Polyakov loop, the quark number density, the chiral condensate.

We evaluated the analytic continuation of these quantities.

Polyakov Loop

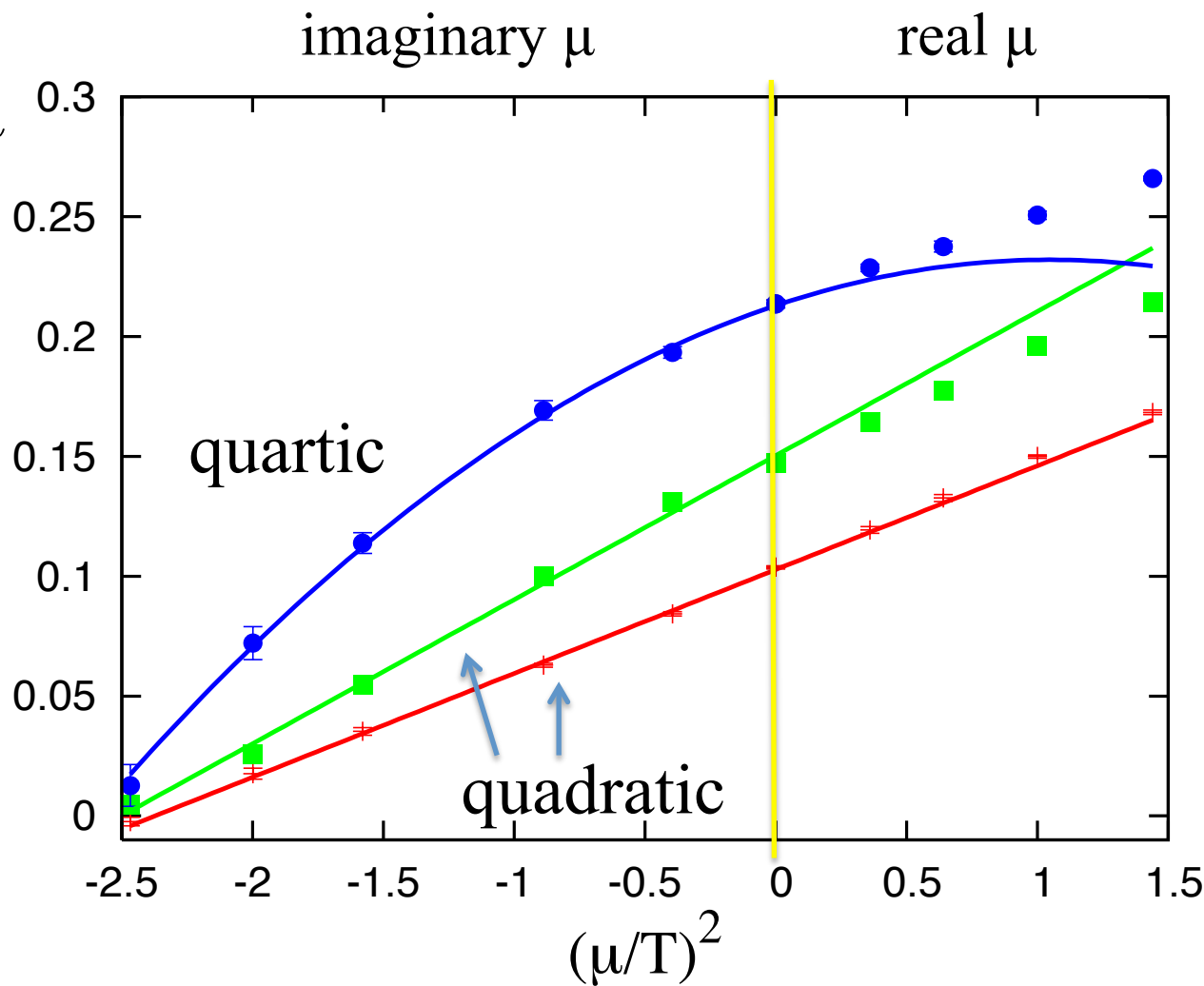
$$L = \frac{1}{N_c} \text{tr}_c \prod_t U_4(\mathbf{x}, t)$$

$$\langle L \rangle = \sum_{n=0}^{n_{\max}} a_{2n} \mu^{2n}$$

$$T/T_c = 0.815$$

$$T/T_c = 0.95$$

$$T/T_c = 1.08$$



fugacity expansion

$$\theta = \mu/T$$

$$Z(\mu) = \sum_{n=-\infty}^{\infty} \underline{Z_n} e^{n\mu/T} = \sum_{k \geq 0} Z_{2k} \cosh(2k\theta)$$

Z_n canonical partition function,

satisfying $Z_n = Z_{-n}$, $Z_{n \neq kN_c} = 0$

Number density

$$\langle n \rangle = T \frac{\partial}{\partial \mu} \ln Z(\mu) = \frac{\sum 2k Z_{2k} \sinh(2k\theta)}{\sum Z_{2k} \cosh(2k\theta)}$$

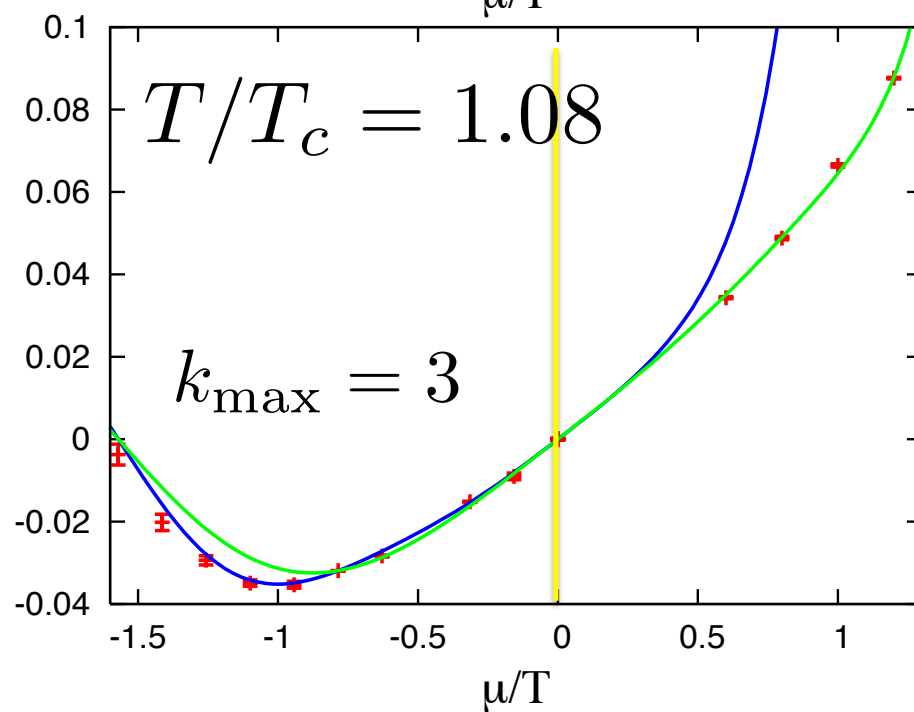
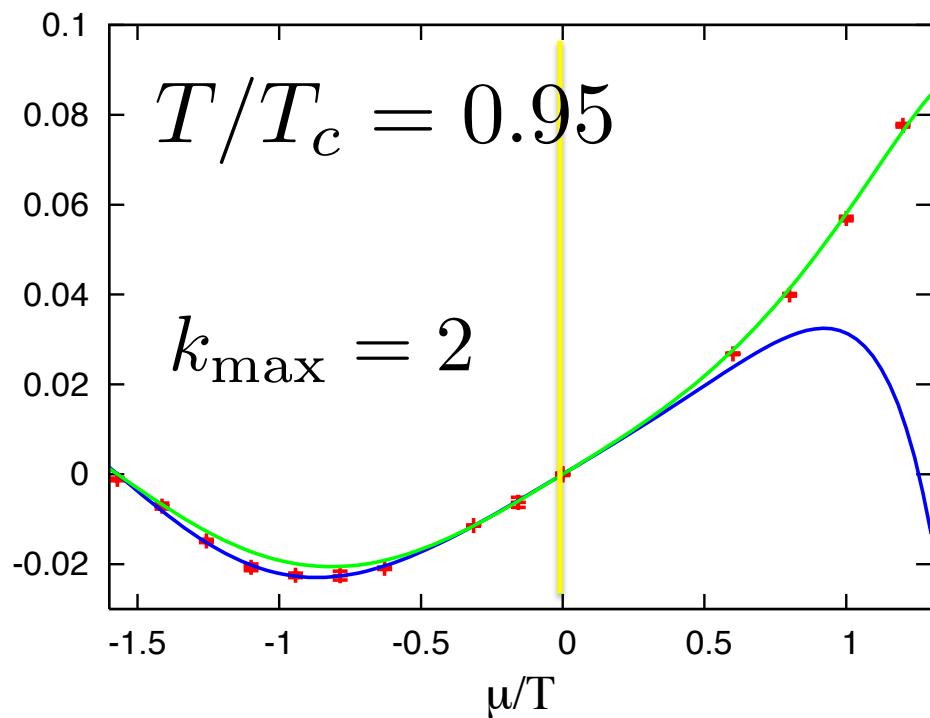
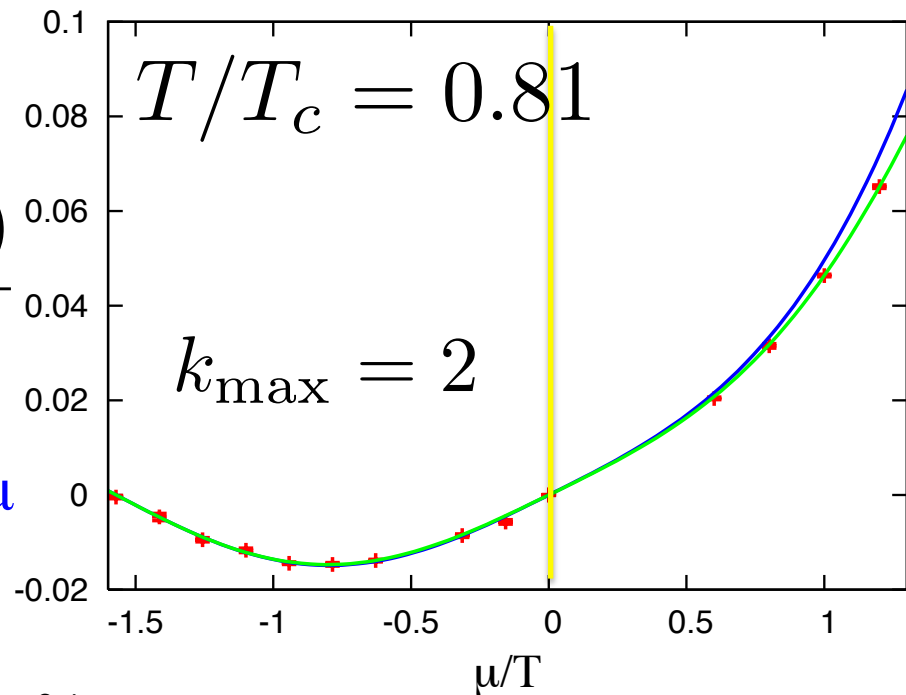
Chiral condensate

$$\langle \bar{q}q \rangle = T \frac{\partial}{\partial m_q} \ln Z(\mu) = \frac{\sum D_{2k} \cosh(2k\theta)}{\sum Z_{2k} \cosh(2k\theta)}, \quad D_{2k} = T \frac{\partial Z_{2k}}{\partial m_q}$$

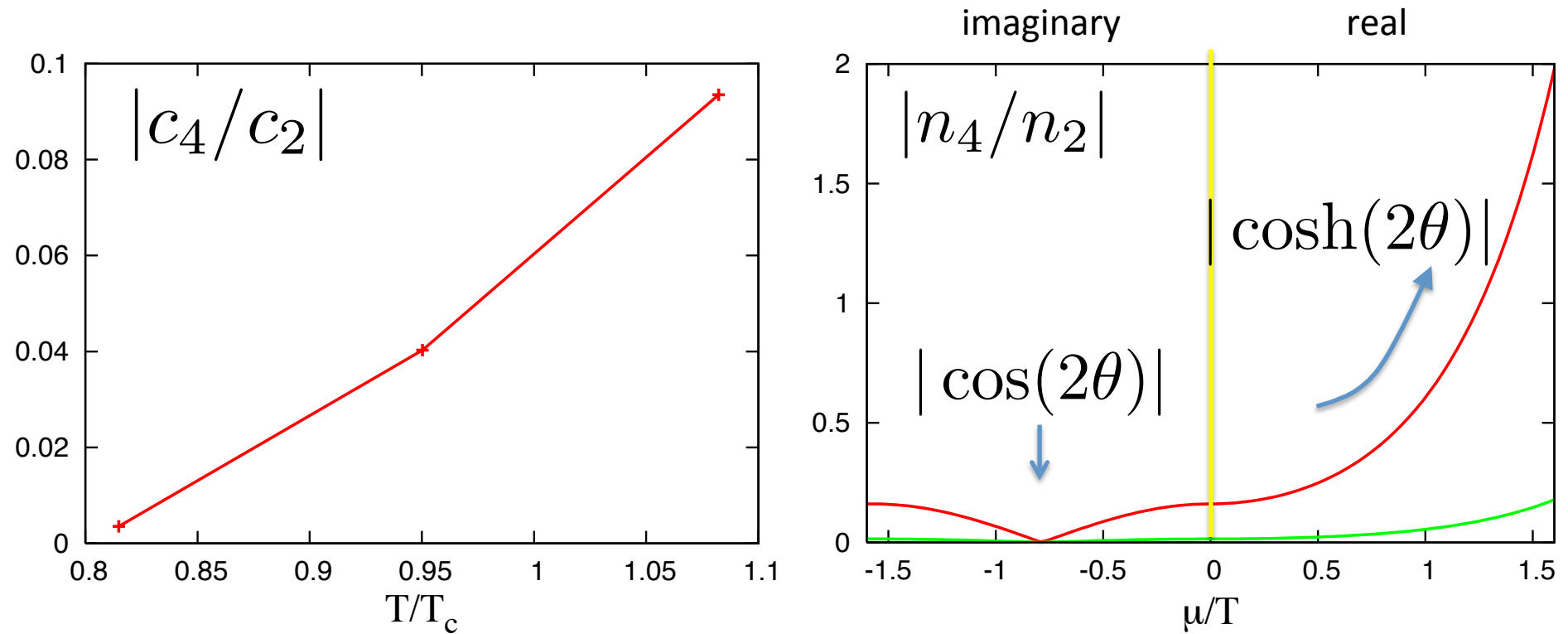
Quark Number Density

$$\langle n \rangle = \frac{\sum_{k \leq k_{\max}} 2k c_{2k} \sinh(2k\theta)}{\sum_{k \leq k_{\max}} c_{2k} \cosh(2k\theta)}$$

Blue : fitted c_{2k} at only imaginary μ
 Green : at all μ region



$$\langle n \rangle = \frac{\sum_k n_{2k}}{Z(\mu)}, \quad n_{2k} = 4k c_{2k} \sinh(2k\theta)$$



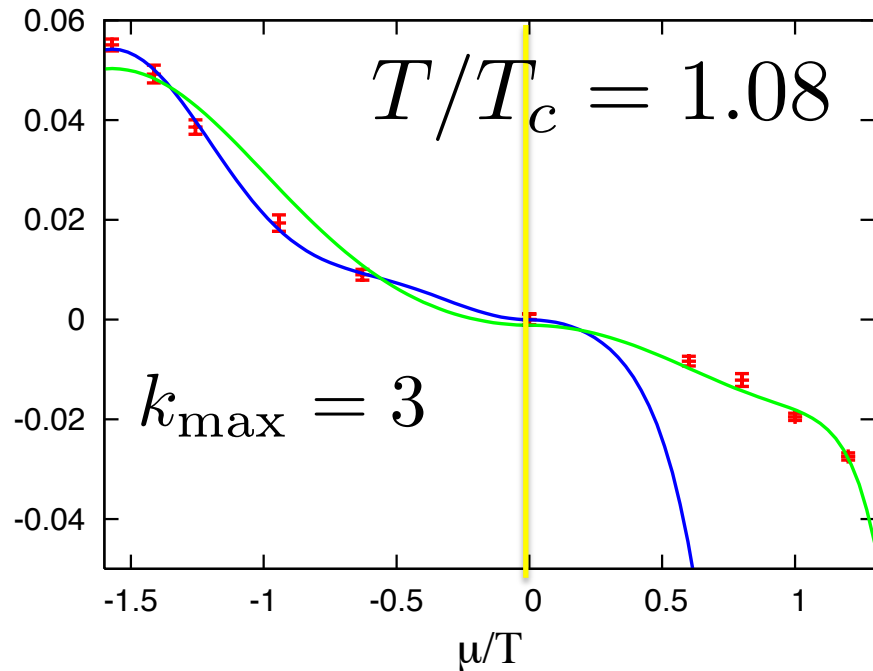
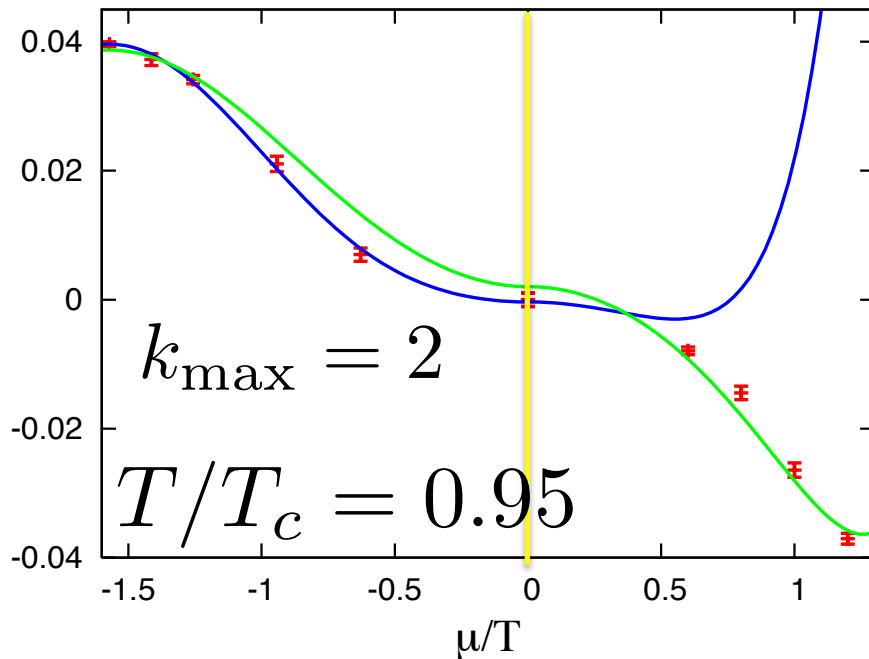
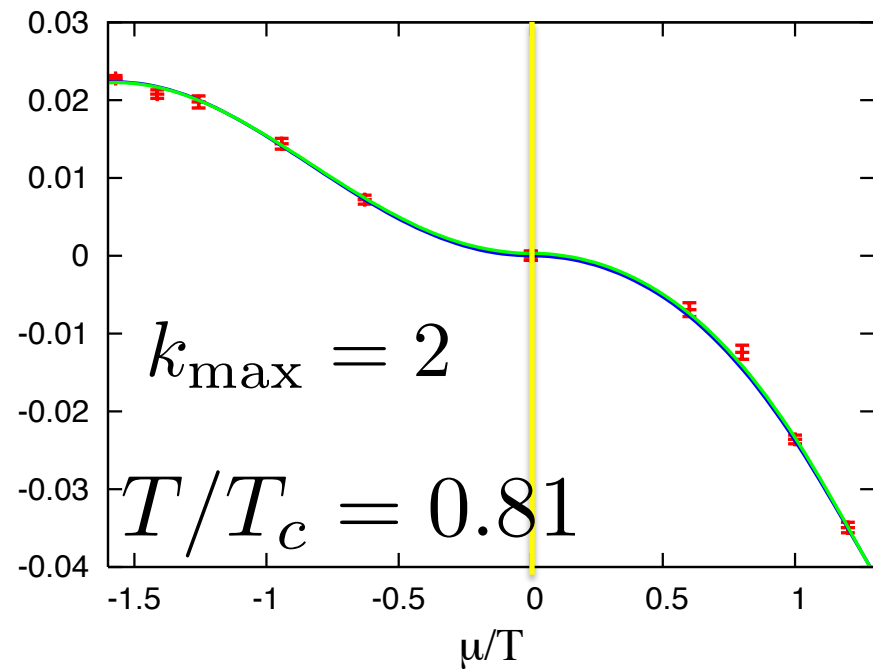
In order to predict large real μ ,
we have to evaluate higher order correction at imaginary μ .

Chiral Condensate

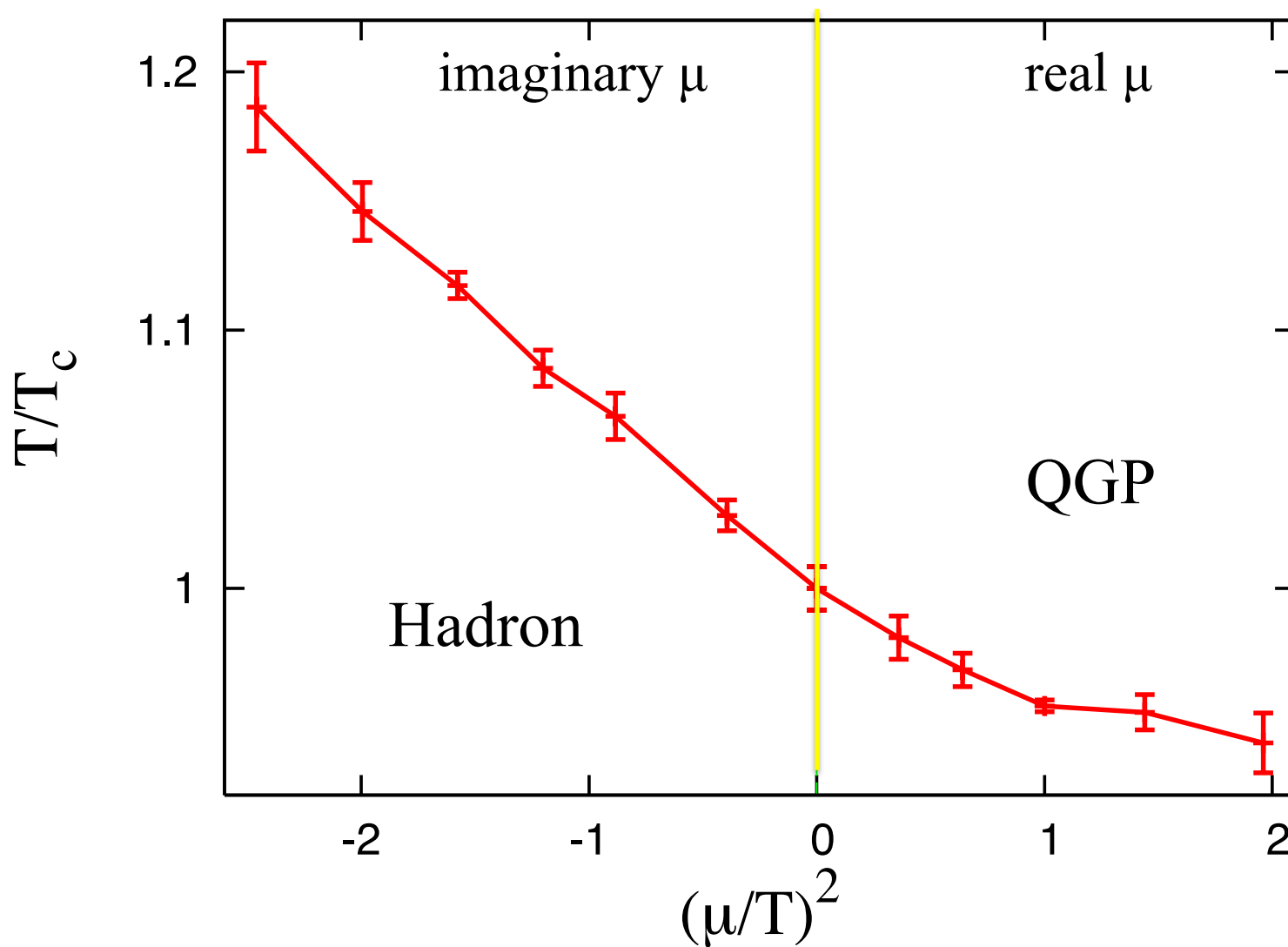
$$\langle \bar{q}q \rangle = \frac{\sum d_{2k} \cosh(2k\theta)}{Z(\mu)}$$

evaluated in n

Blue : fitted d_{2k} at only imaginary μ
 Green : at all μ region

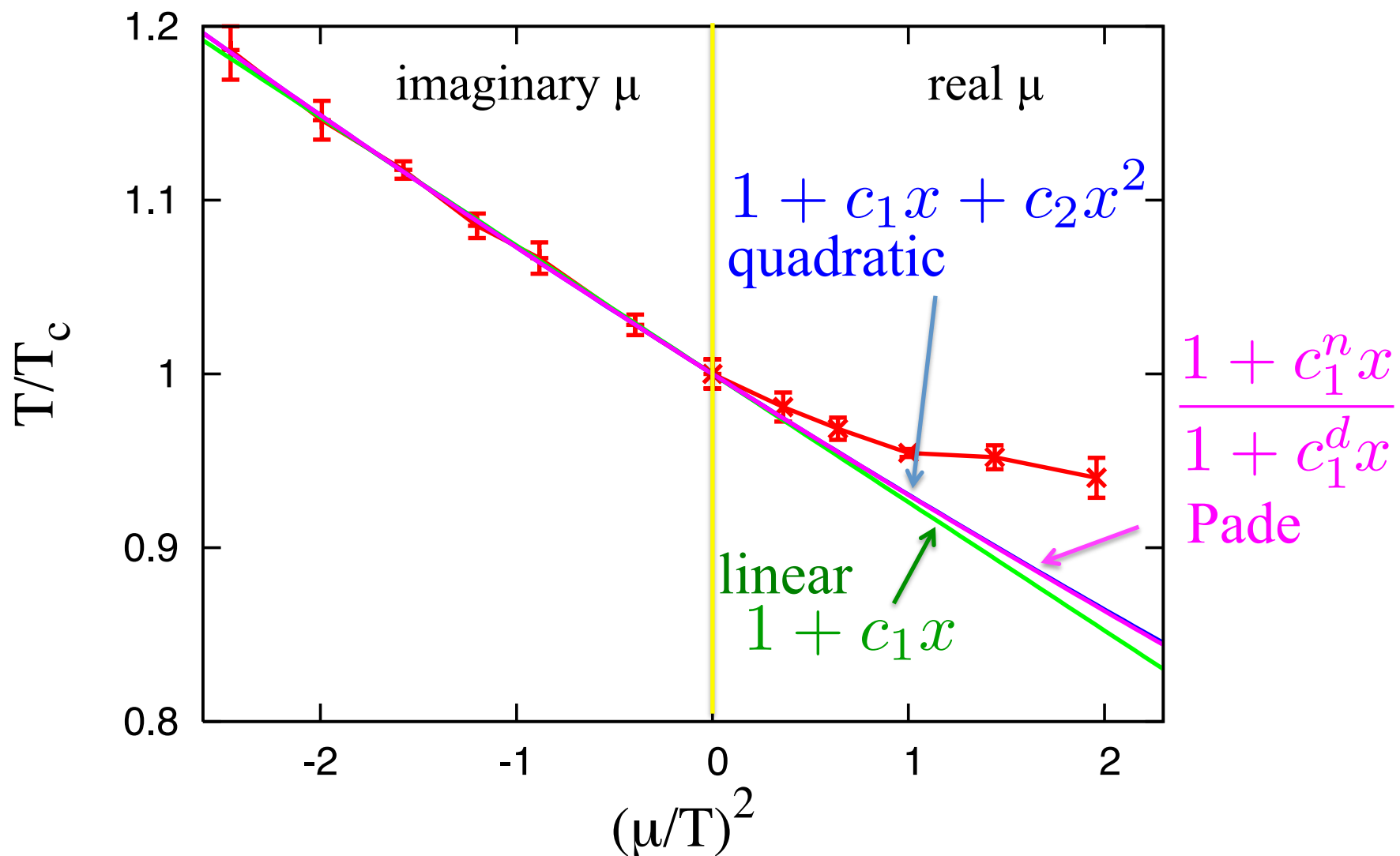


Phase Transition Line

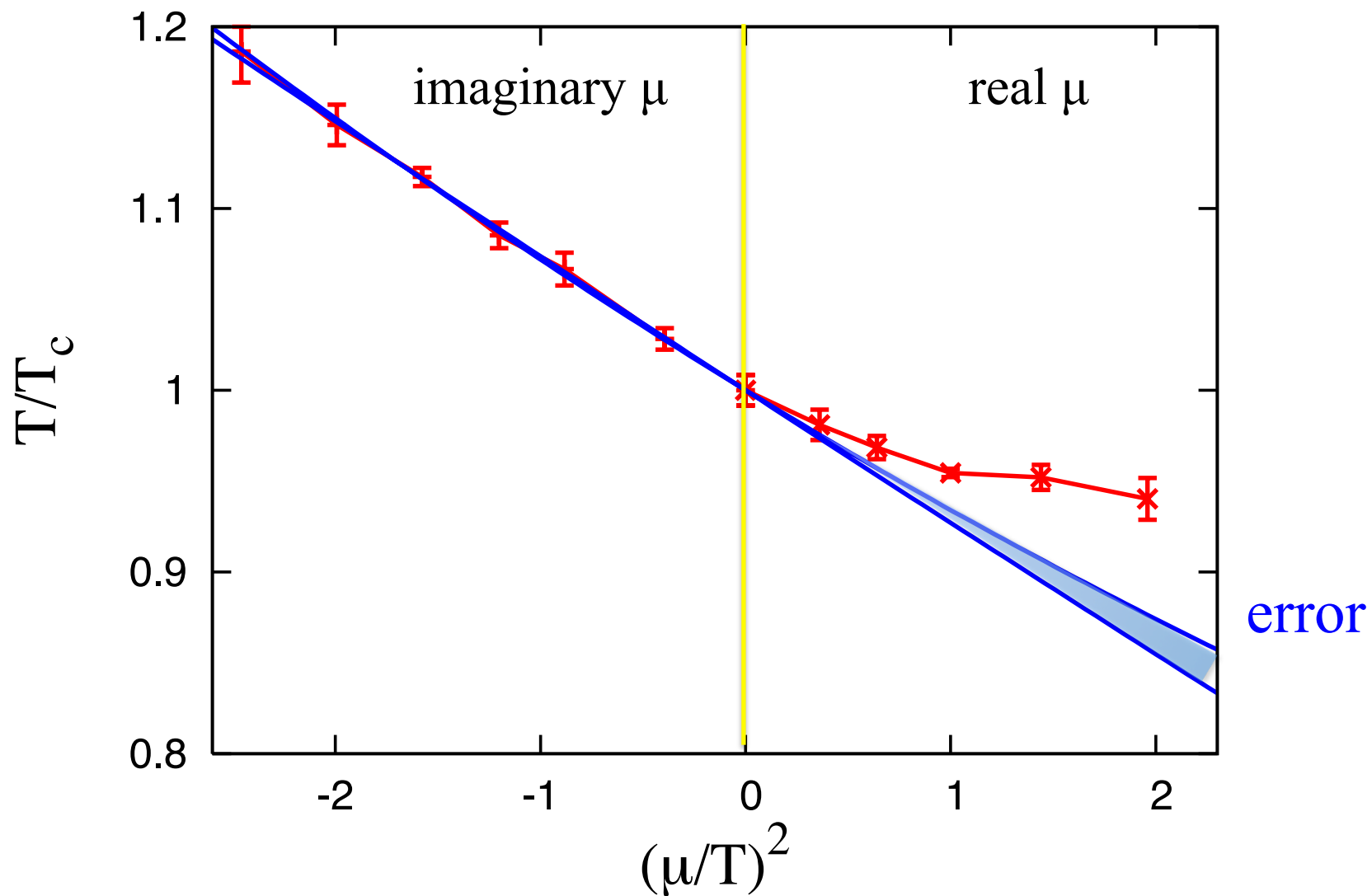


Phase Transition Line

$$x = (\mu/T)^2$$



Phase Transition Line



Summary

We calculated 2-color QCD phase diagram
with RG improved gauge + clover improved Wilson fermion.

We checked analytic continuation from imaginary to real μ .

For some observables, the fitted functions at imaginary μ
can reproduce the exact results well below T_c ,
but the agreement becomes worse above T_c .

For the phase transition line,
the data at imaginary μ is almost on a linear function,
but the function deviates from the exact data at real μ .

Spatial Correlation

Meson Screening Masses
Spatial String Tension

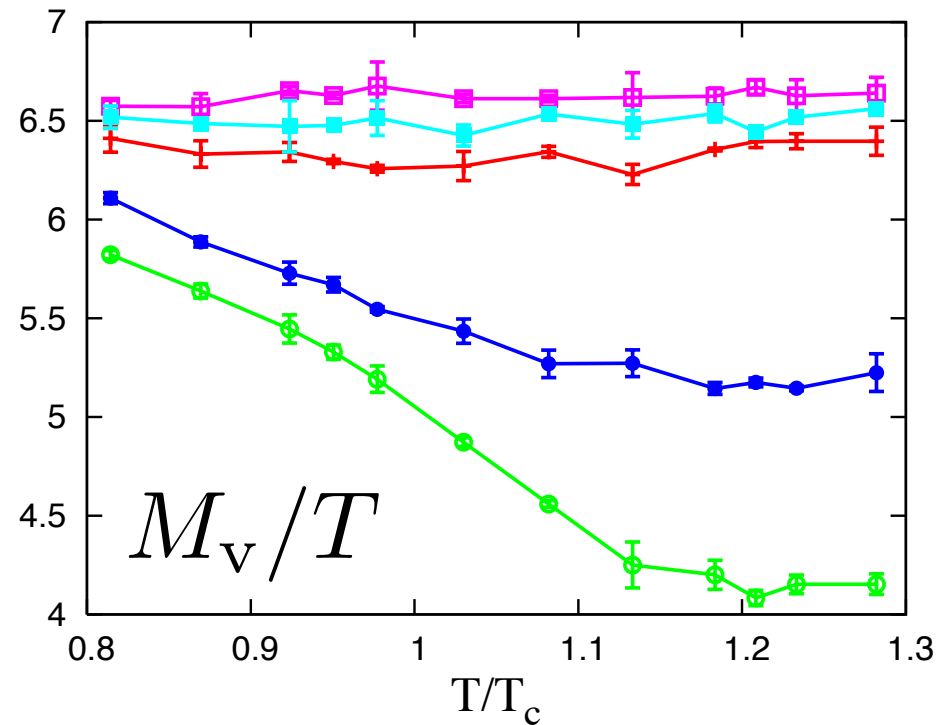
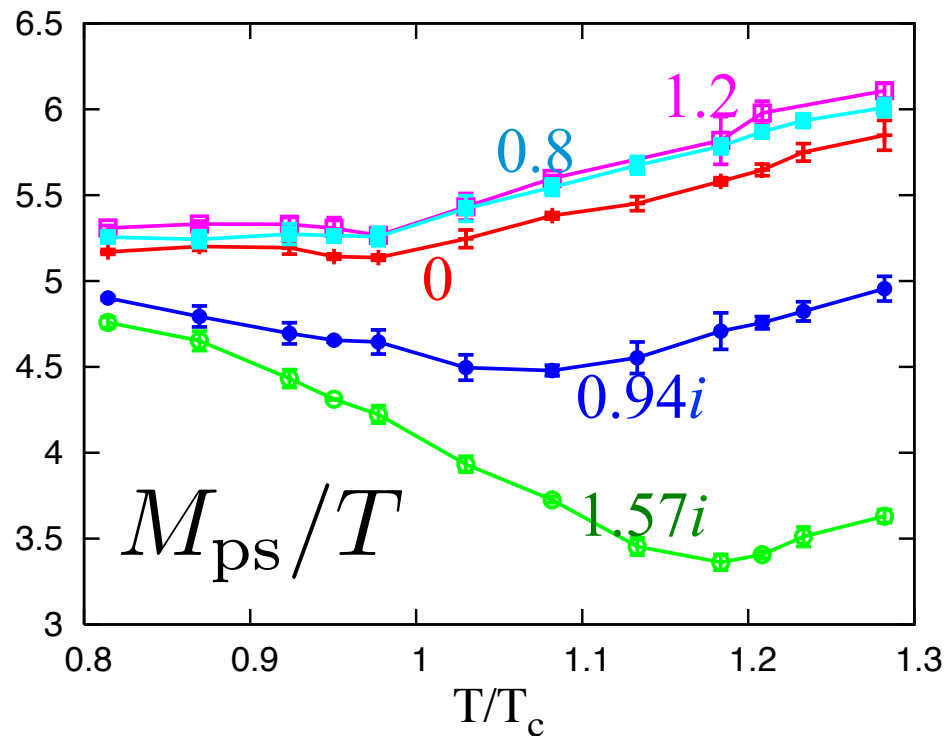
Lattice Setup: $16^3 \times 4$, on the line of the constant physics $M_{ps}/M_v=0.8$

Meson Screening Masses

Preliminary

$$\langle M^\dagger(\mathbf{x}) M(\mathbf{0}) \rangle \sim e^{-M|\mathbf{x}|}$$

The screening masses have strong T dependence at imaginary μ

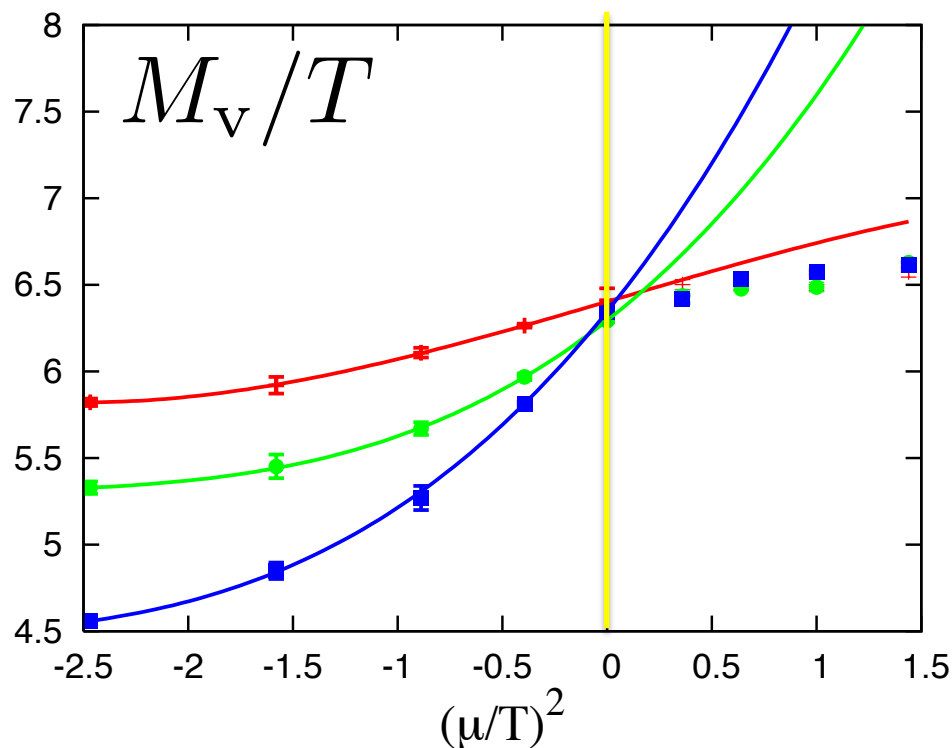
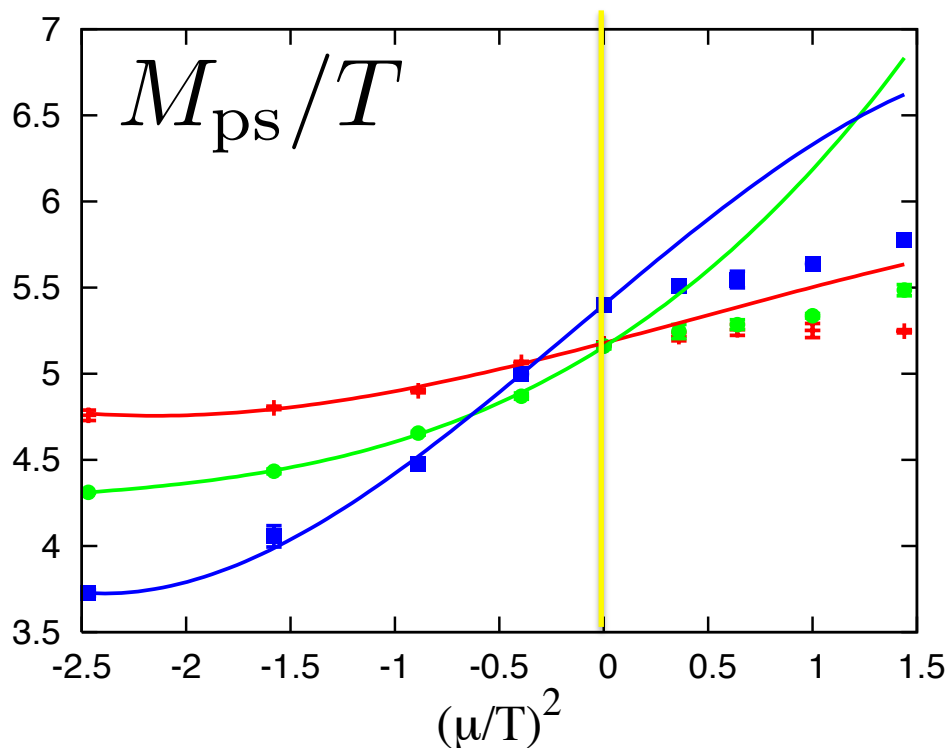


Meson Screening Masses

Preliminary

$$M/T = \sum_{n \leq 3} a_n (\mu/T)^{2n}$$

$$T/T_c = 0.81, 0.95, 1.08$$

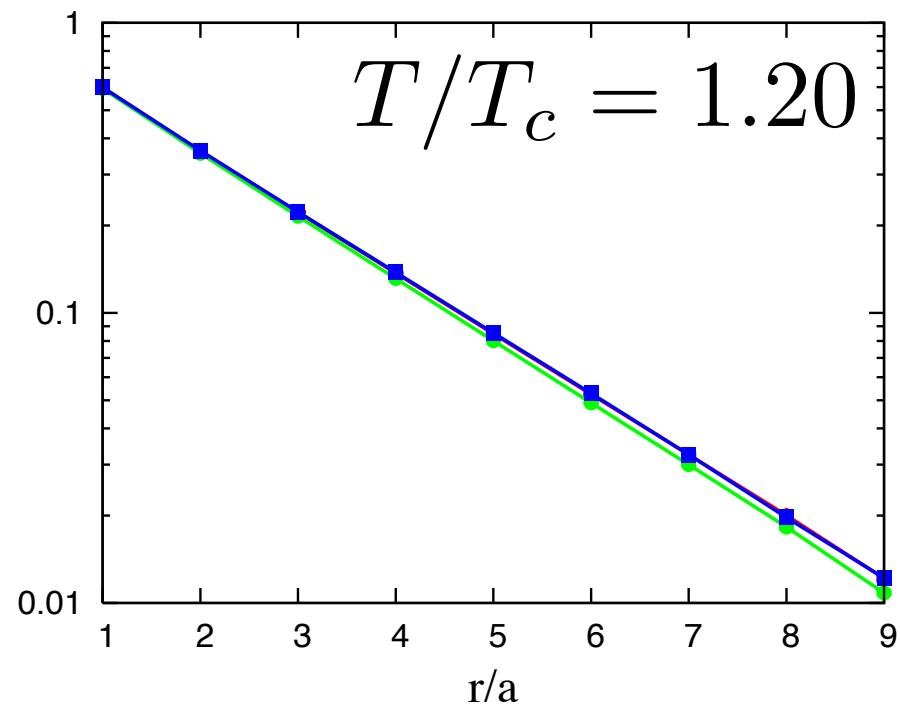
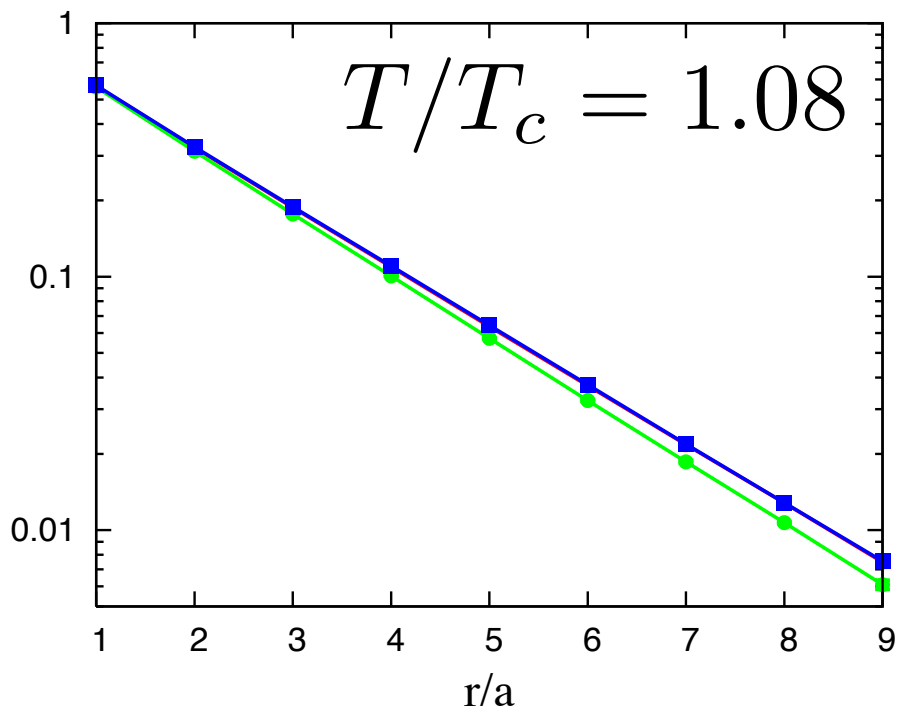
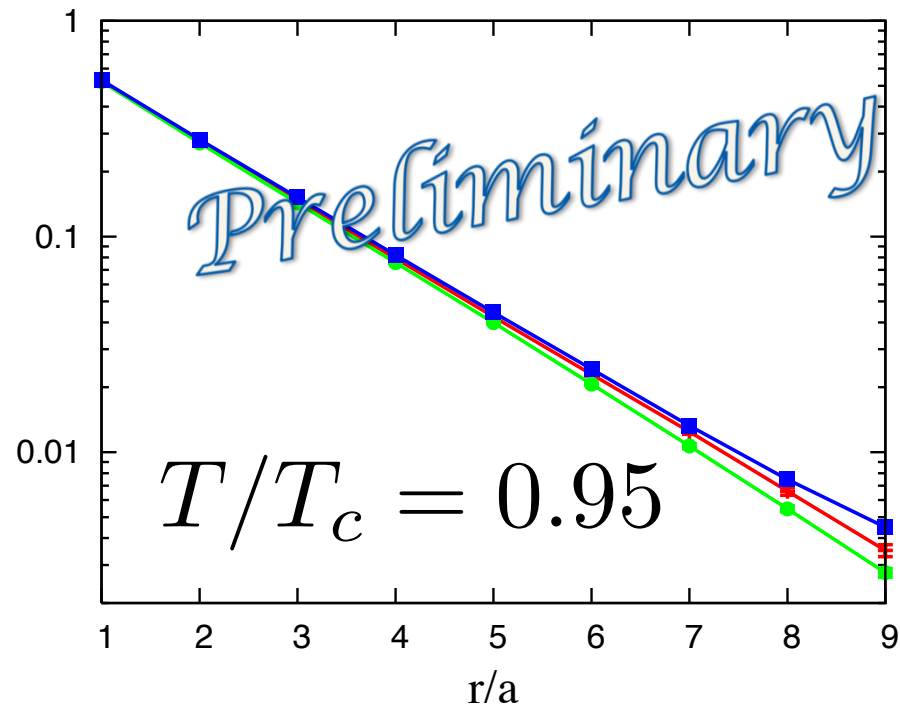


Spatial Wilson Loop

$$W(x, y) \sim \begin{array}{|c|} \hline y \\ \hline \square \\ \hline x \\ \hline \end{array}$$

$$\mu/T = 0, 1.57i, 1.2$$

Also confinement at finite μ



Spatial String Tension

Preliminary

$$-\ln W(x, y) = \sigma_s xy + \sigma_p(x + y) + c$$

σ_s also has stronger imaginary μ dependence than imaginary μ

