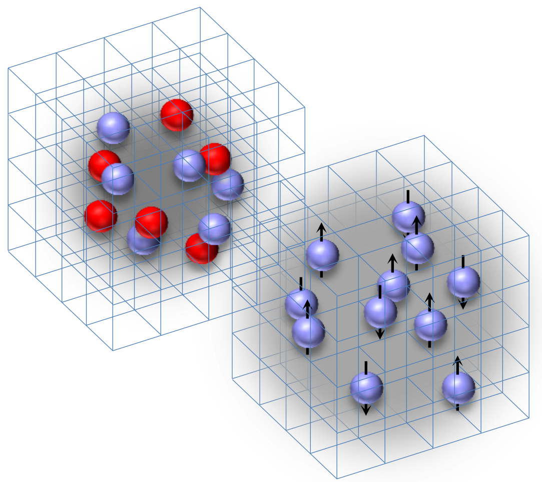
Lattice Effective Field Theory for Nuclear Physics



Nuclear Lattice EFT Collaboration

Evgeny Epelbaum (Bochum)
Hermann Krebs (Bochum)
Timo Lähde (Jülich)
Dean Lee (NC State)
Ulf-G. Meißner (Bonn/Jülich)
Gautam Rupak (MS State)

XQCD13
Workshop on QCD under extreme conditions
Bern, Switzerland
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Outline

Introduction and motivation

What is lattice effective field theory?

Lattice results for light nuclei

Carbon-12 spectrum and the Hoyle state

Light quark mass dependence of helium burning

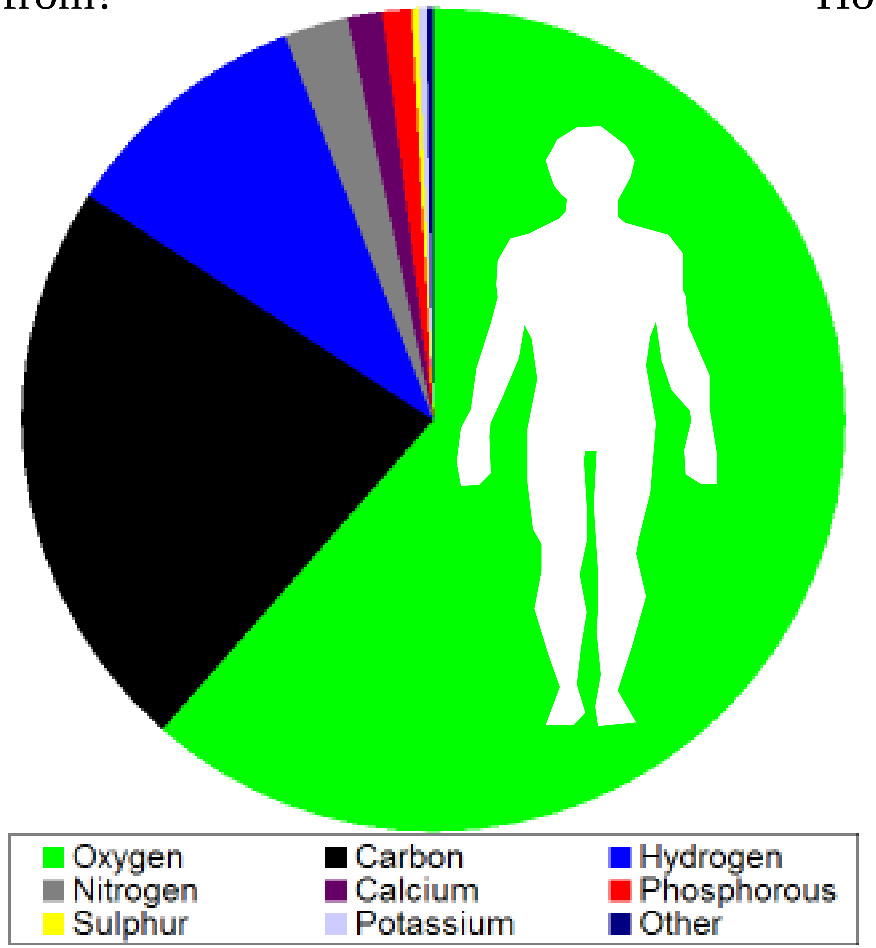
Scattering and reactions on the lattice

Summary and future directions

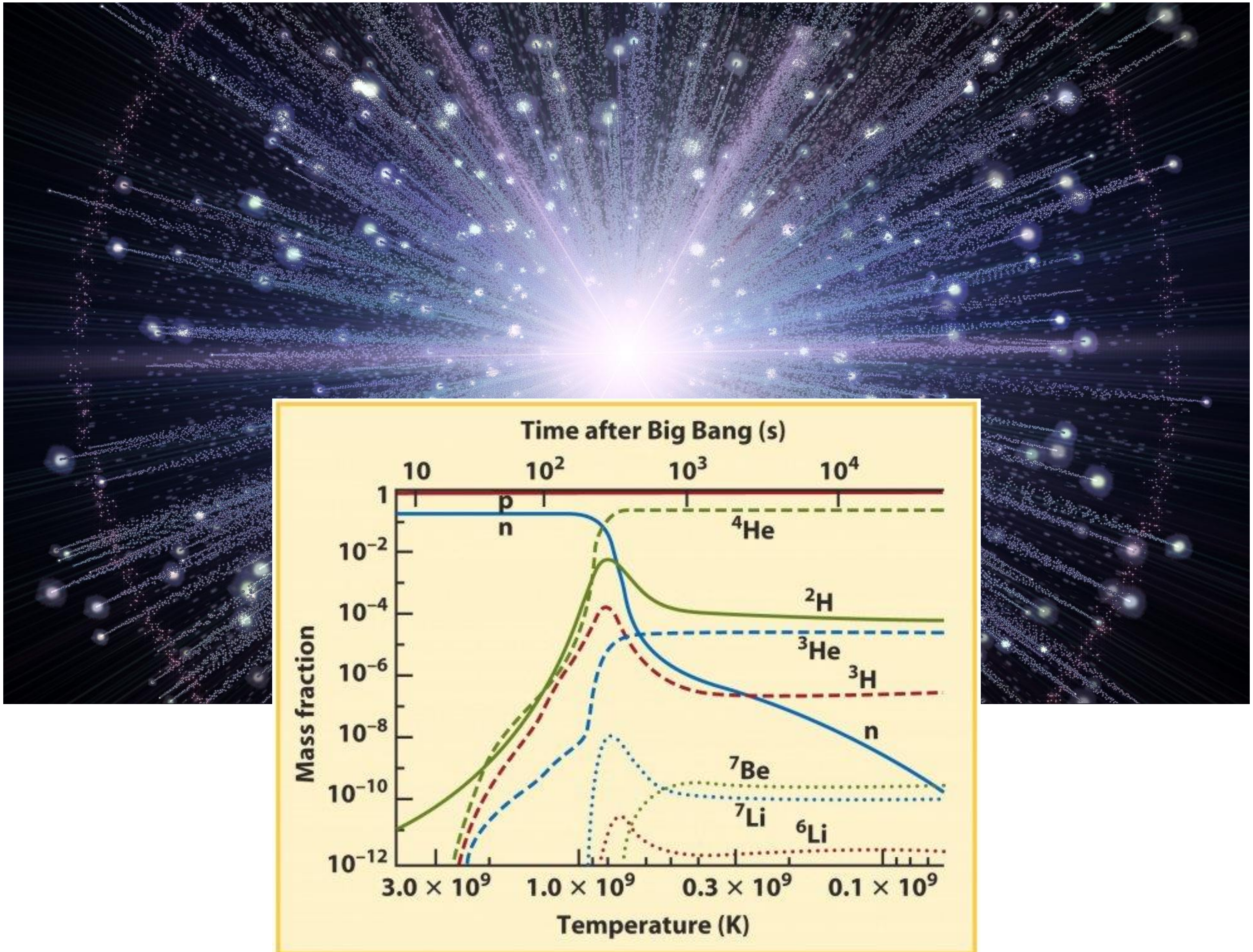
Elemental composition of the human body by mass

Where did it come from?

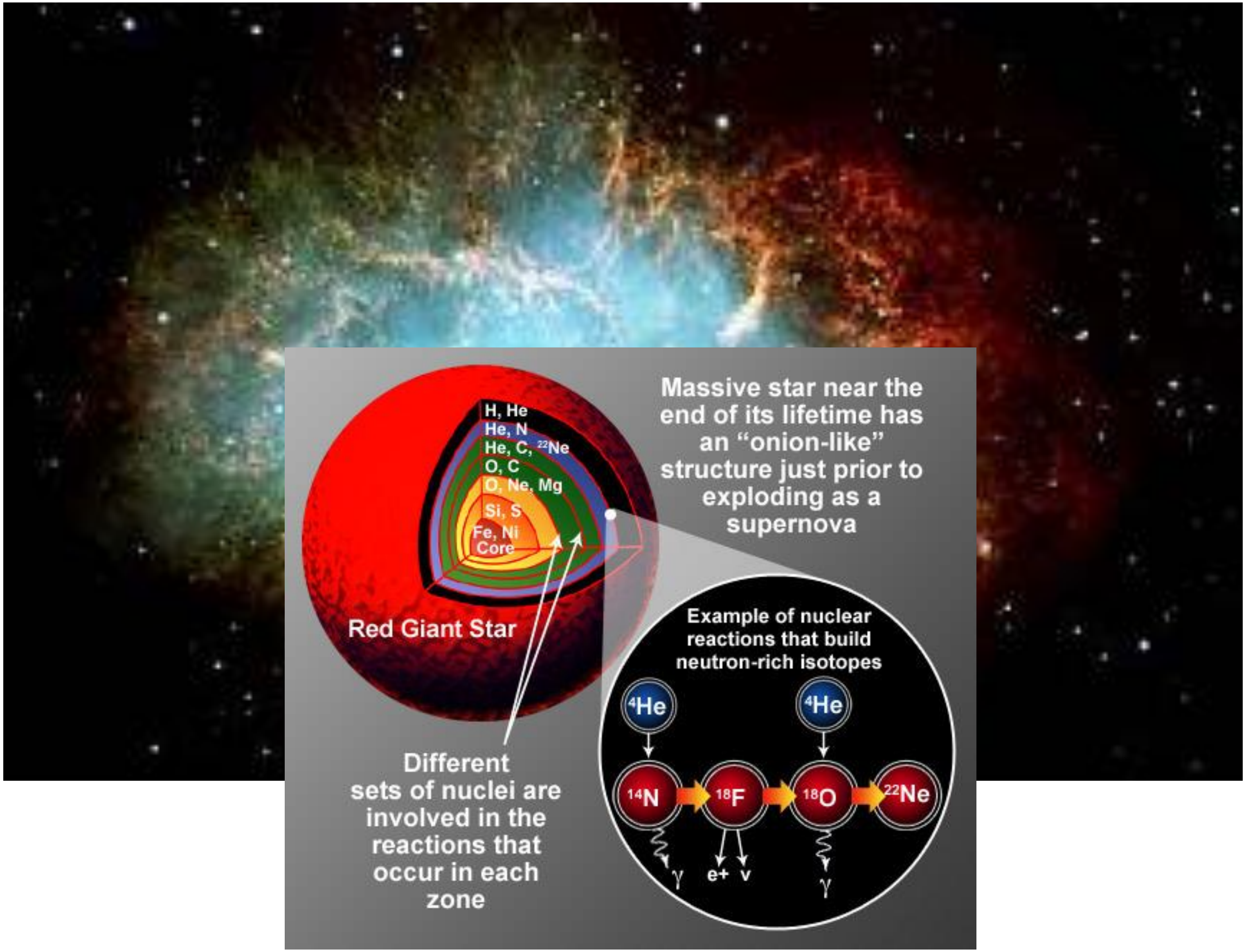
How was it made?



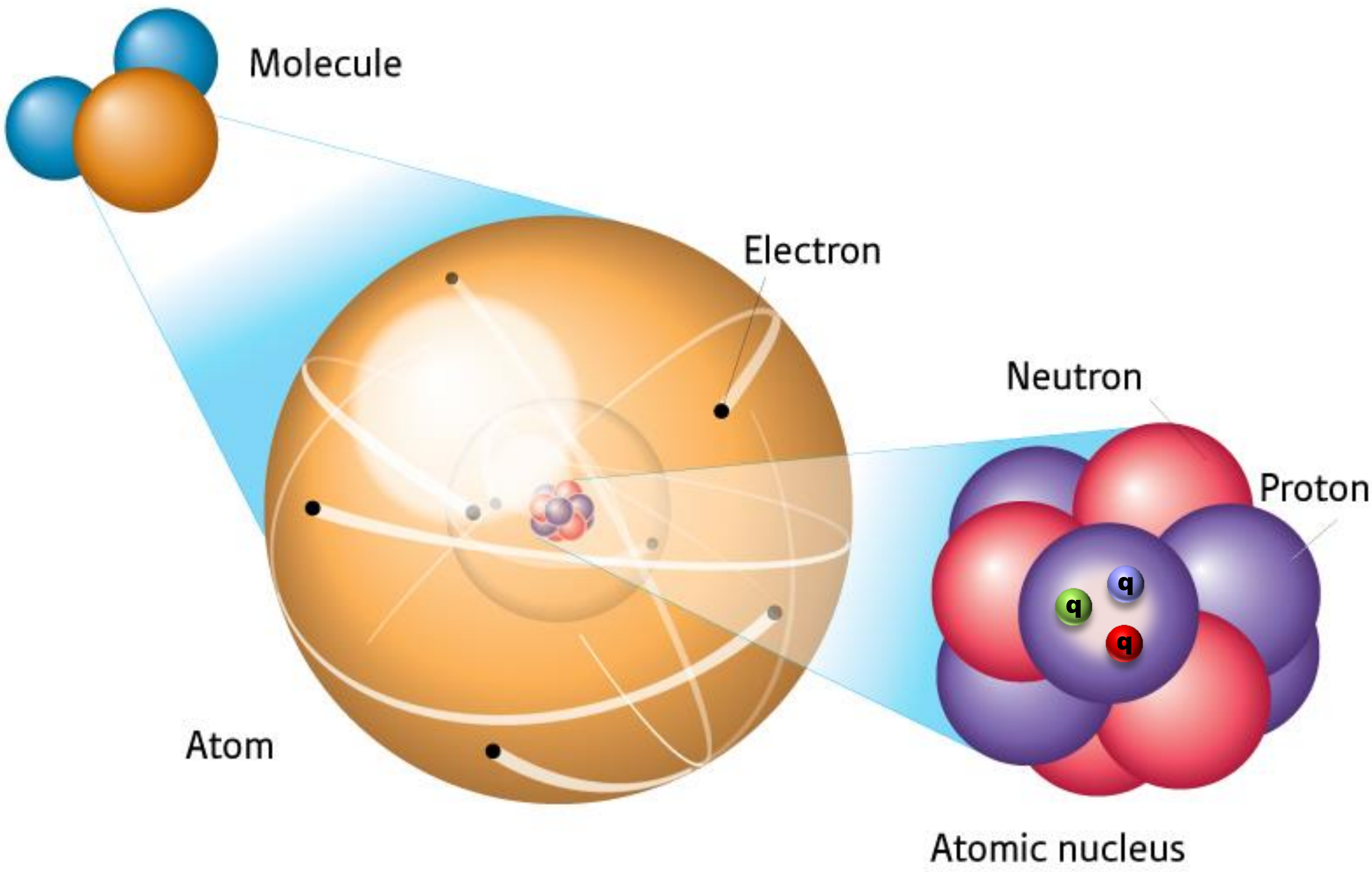
Big bang nucleosynthesis



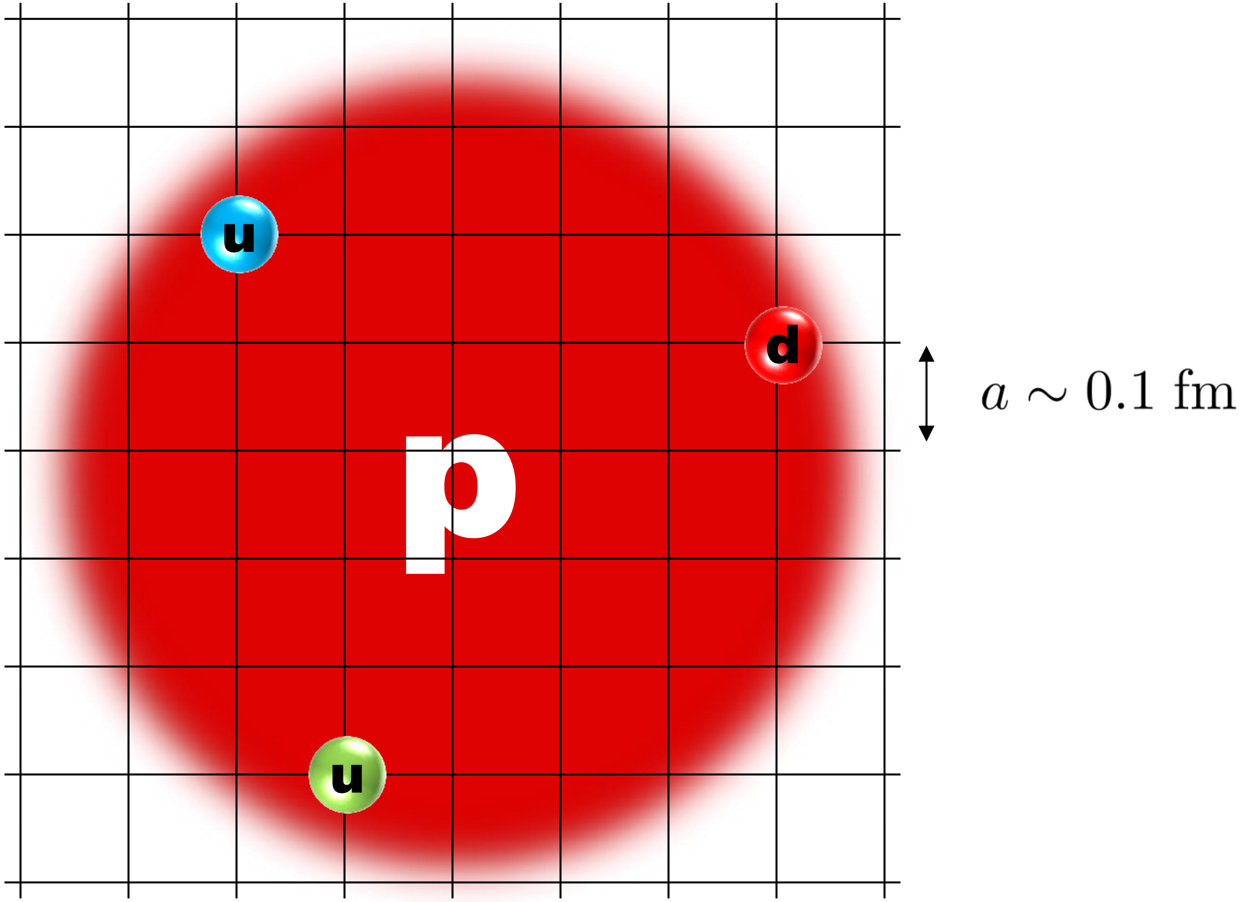
Stellar nucleosynthesis



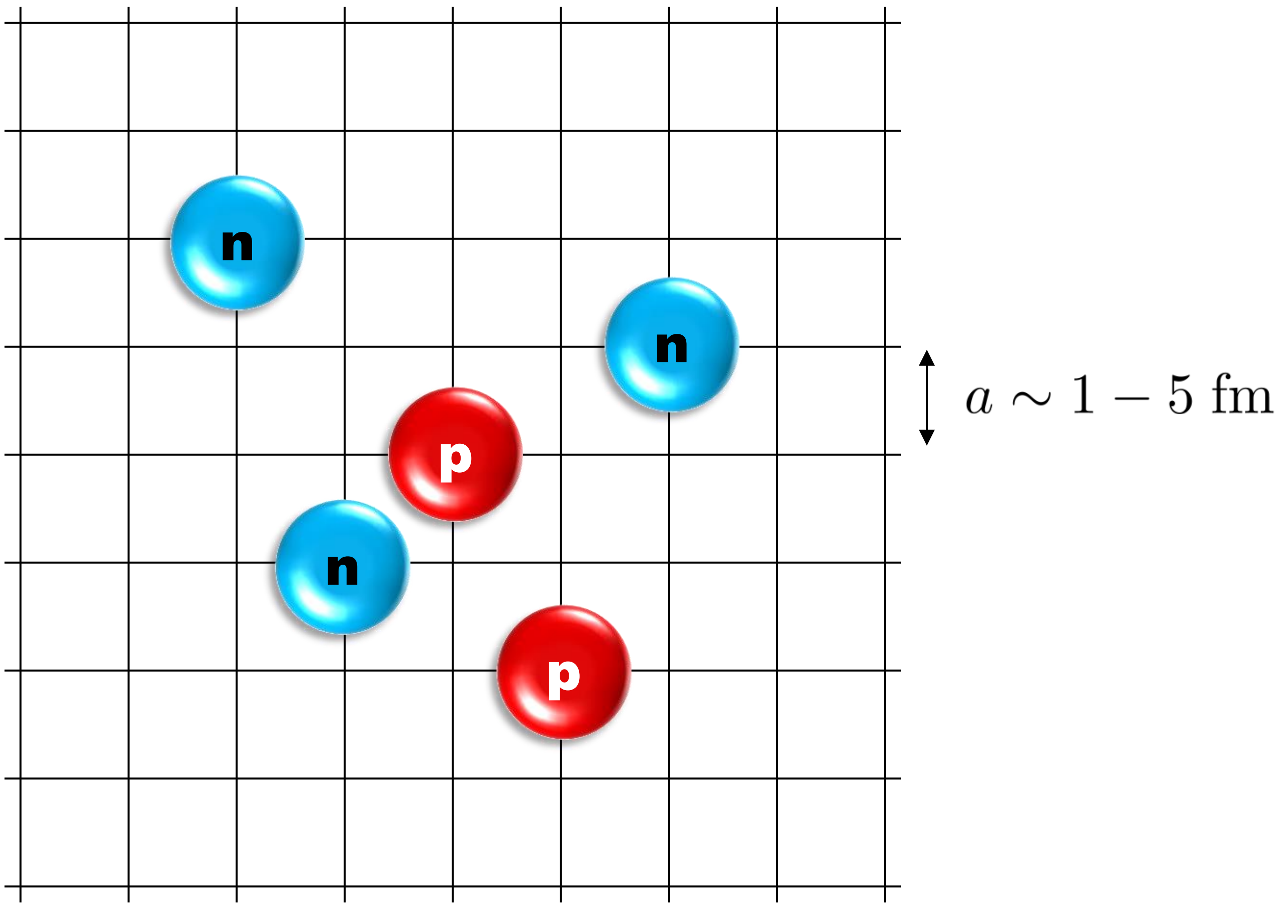
Simulation as the next generation microscope

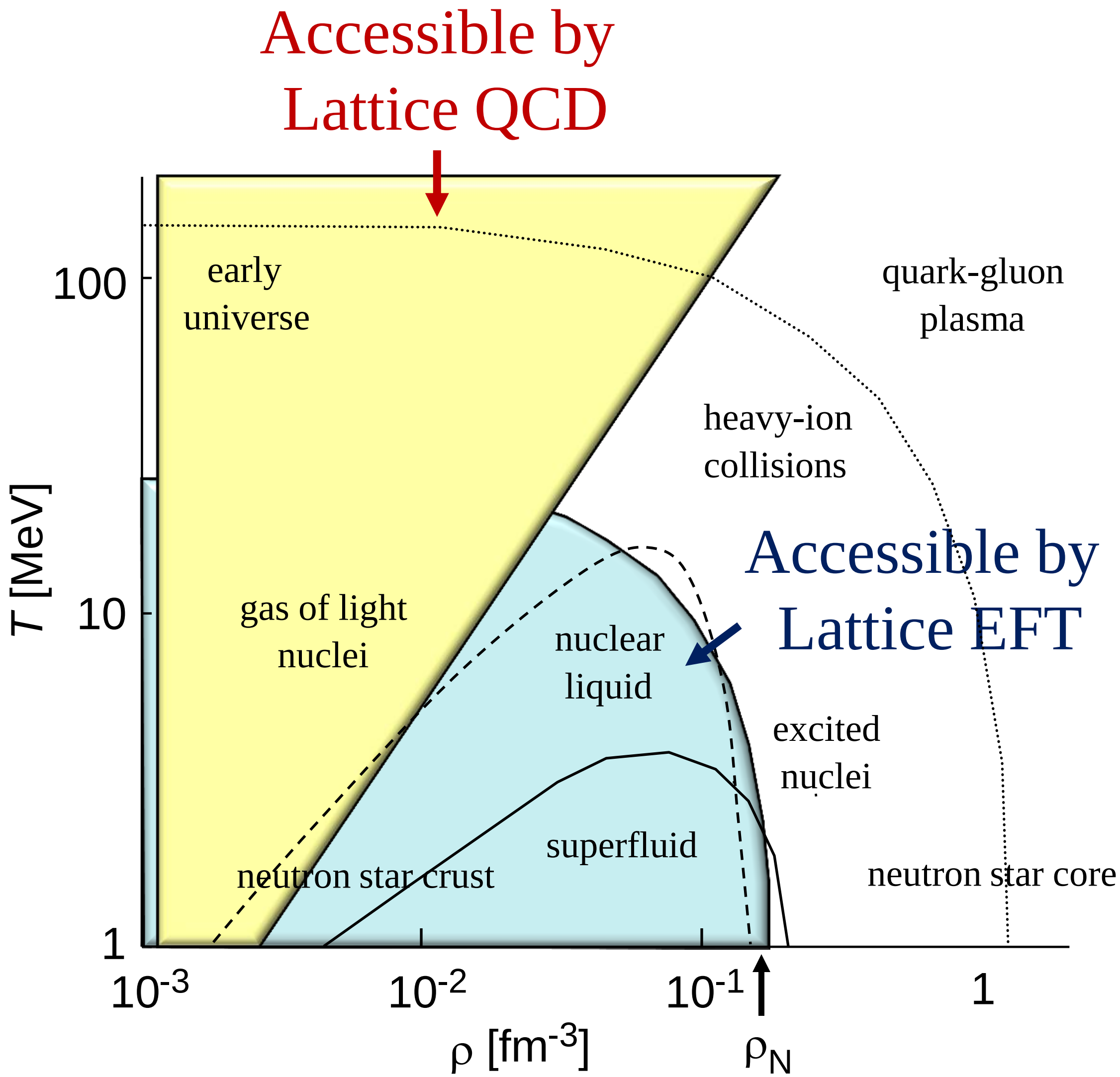


Lattice quantum chromodynamics



Lattice effective field theory




Accessible by
Lattice QCD
100
10
1
T [MeV]
early
universe
quark-gluon
plasma
heavy-ion
collisions
gas of light
nuclei
nuclear
liquid
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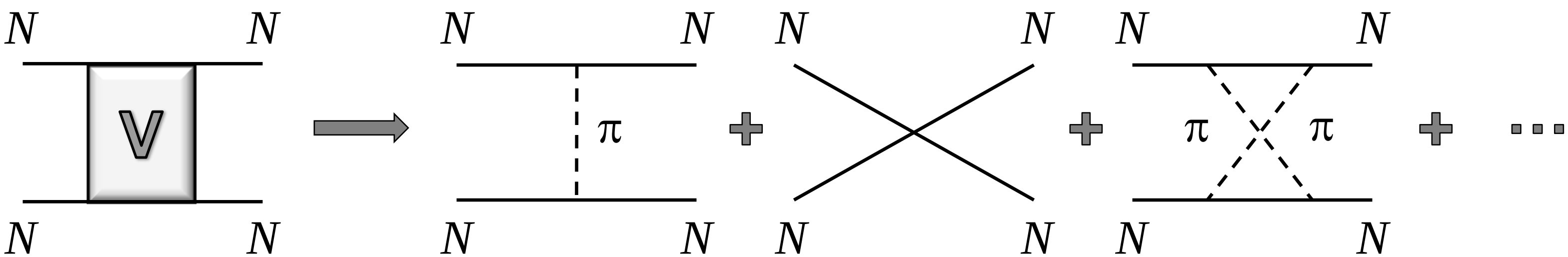
Lattice EFT
excited
nuclei
superfluid
neutron star crust
neutron star core
10-3
10-2
10-1
1
ρ [fm-3]
ρN

Low energy nucleons: Chiral effective field theory

Construct the effective potential order by order



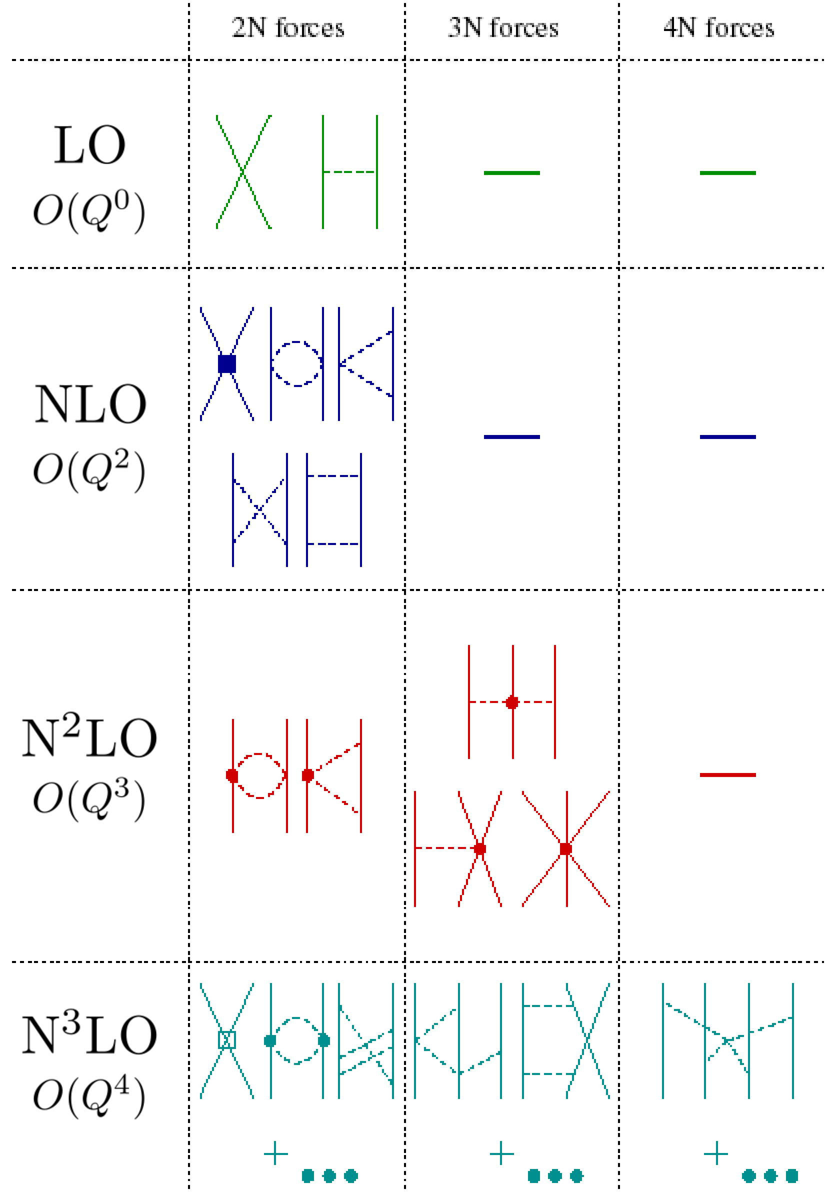
$$\underbrace{V^{\mathrm{OPEP}} \quad \text{Contact interactions}}_{\text{Leading order (LO)}} \qquad \underbrace{V^{\mathrm{TPEP}}}_{\text{Next-to-leading order (NLO)}}$$

Nuclear Scattering Data

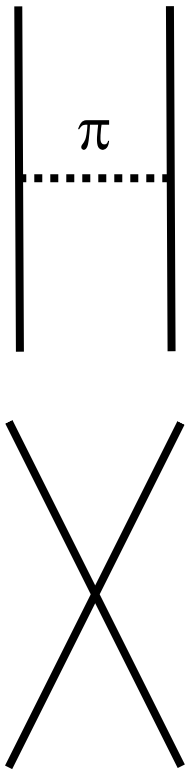
↓

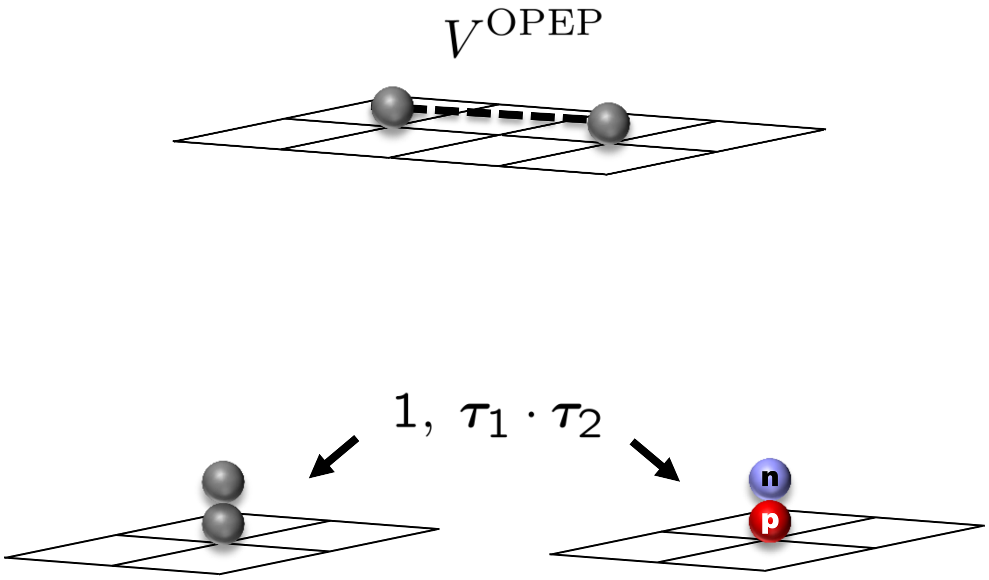
Effective Field Theory

Ordonez et al. ’94; Friar & Coon ’94; Kaiser et al. ’97; Epelbaum et al. ’98, ‘03; Kaiser ’99-’01; Higa et al. ’03; …

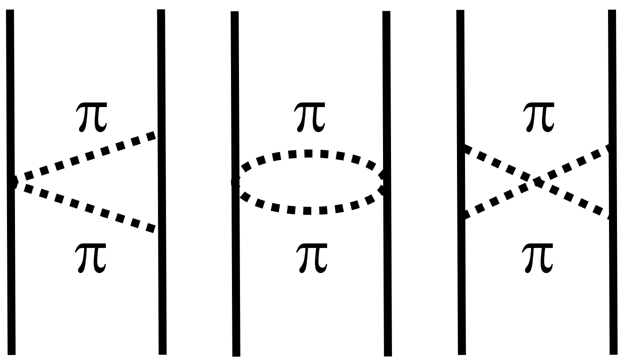


Leading order on the lattice

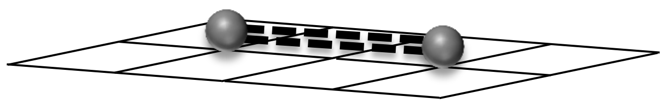


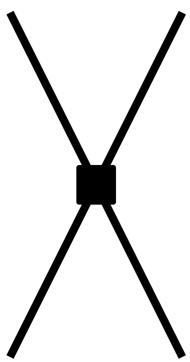


Next-to-leading order on the lattice

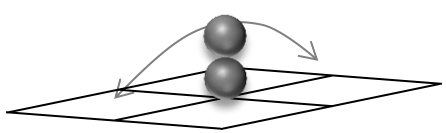


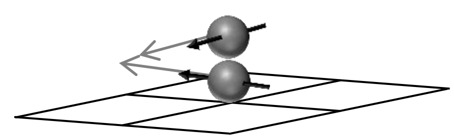
V^{TPEP}





$\vec{\nabla}_1 \cdot \vec{\nabla}_2 \qquad (\vec{\sigma}_1 \cdot \vec{\nabla}_1)\,(\vec{\sigma}_2 \cdot \vec{\nabla}_2) \qquad \cdots$



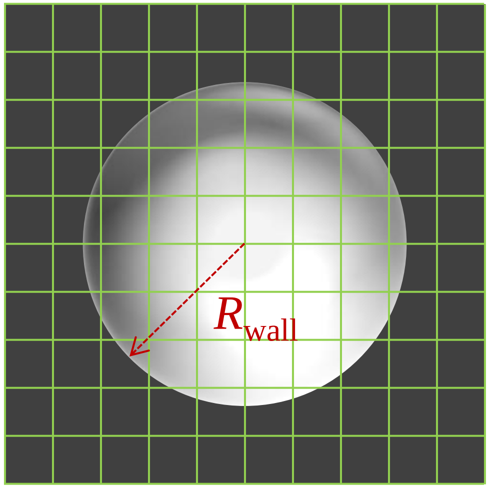


Physical scattering data → Unknown operator coefficients

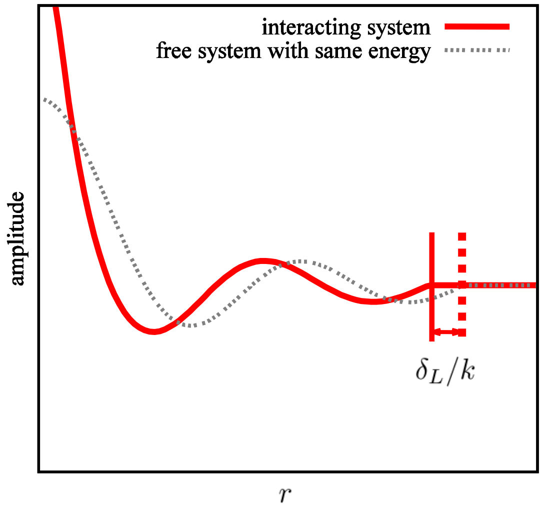
Spherical wall method

Borasoy, Epelbaum, Krebs, D.L., Meißner, EPJA 34 (2007) 185

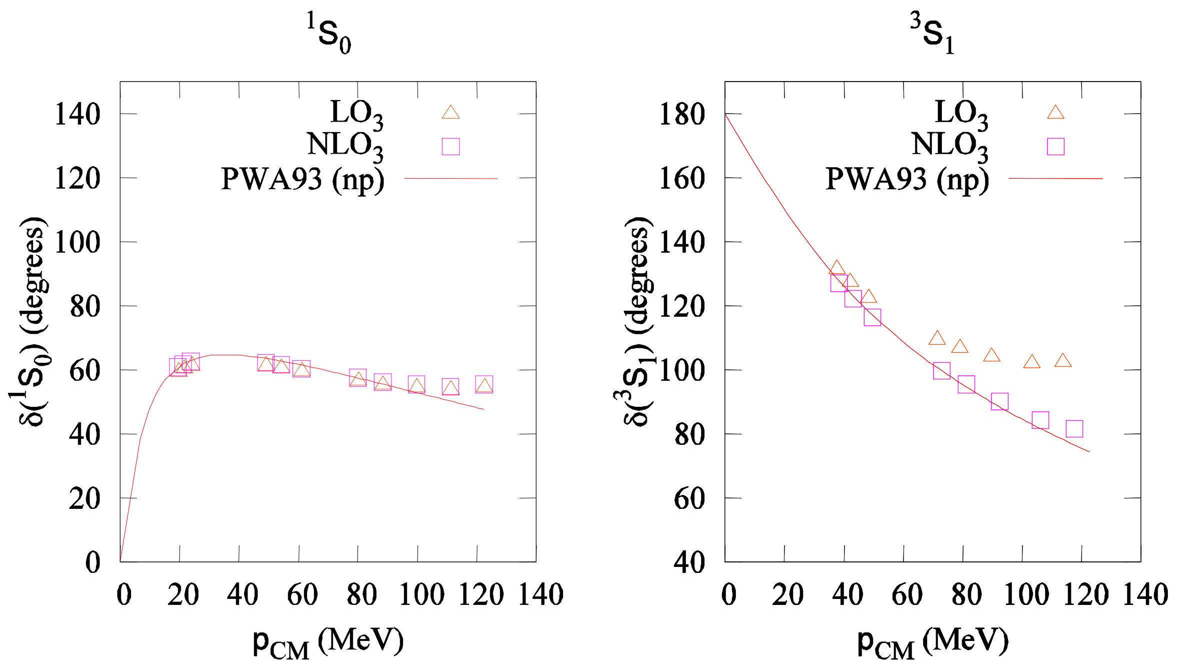
Spherical wall imposed in the center of mass frame



Representation	J_z	Example
A_1	$0 \bmod 4$	$Y_{0,0}$
T_1	$0,1,3 \bmod 4$	$\{Y_{1,0}, Y_{1,1}, Y_{1,-1}\}$
E	$0,2 \bmod 4$	$\left\{Y_{2,0}, \frac{Y_{2,-2}+Y_{2,2}}{\sqrt{2}}\right\}$
T_2	$1,2,3 \bmod 4$	$\left\{Y_{2,1}, \frac{Y_{2,-2}-Y_{2,2}}{\sqrt{2}}, Y_{2,-1}\right\}$
A_2	$2 \bmod 4$	$\frac{Y_{3,2}-Y_{3,-2}}{\sqrt{2}}$

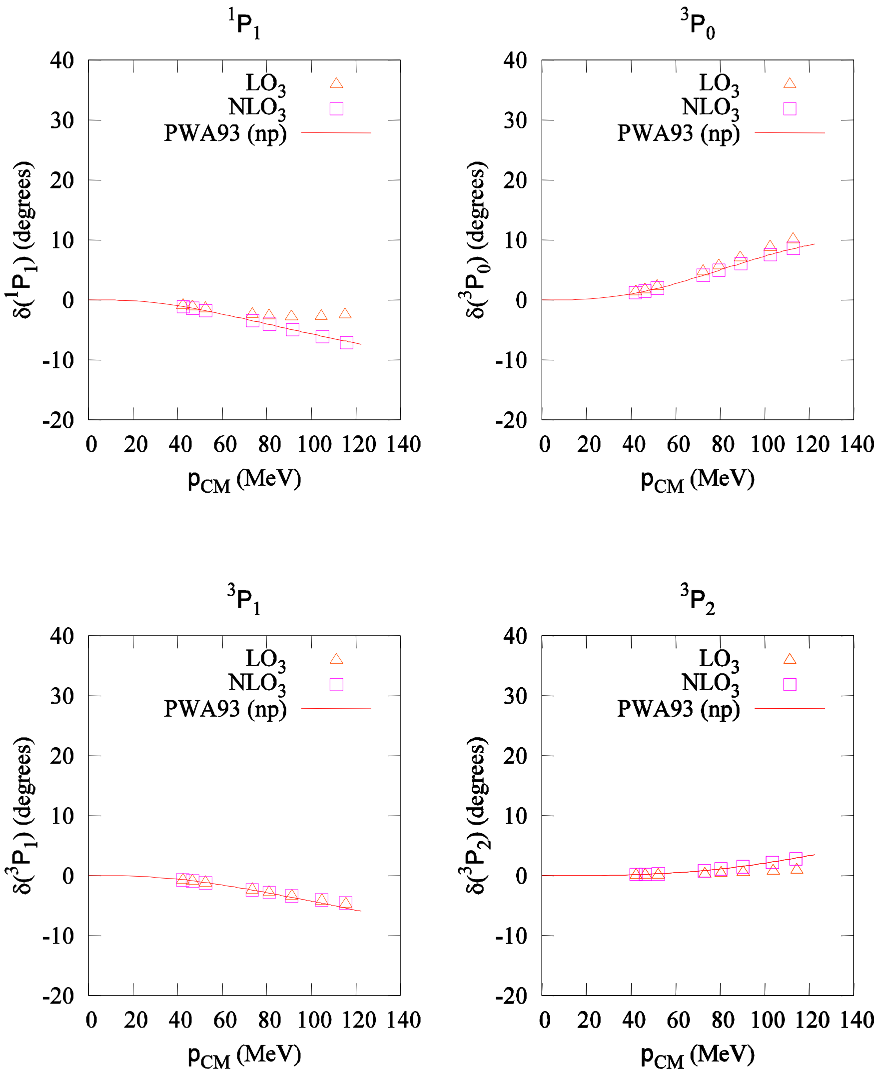


$\underline{LO_3\text{: S waves}}$ $a = 1.97$ fm



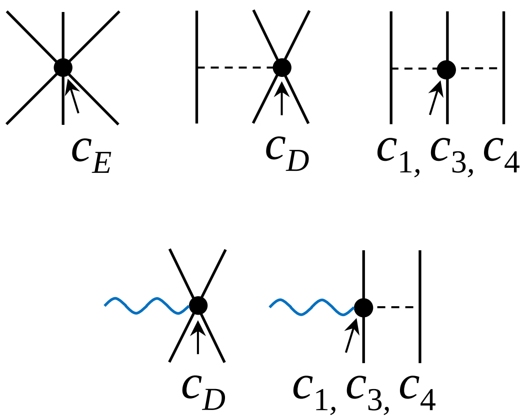
LO$_3$: P waves

$$a = 1.97 \text{ fm}$$

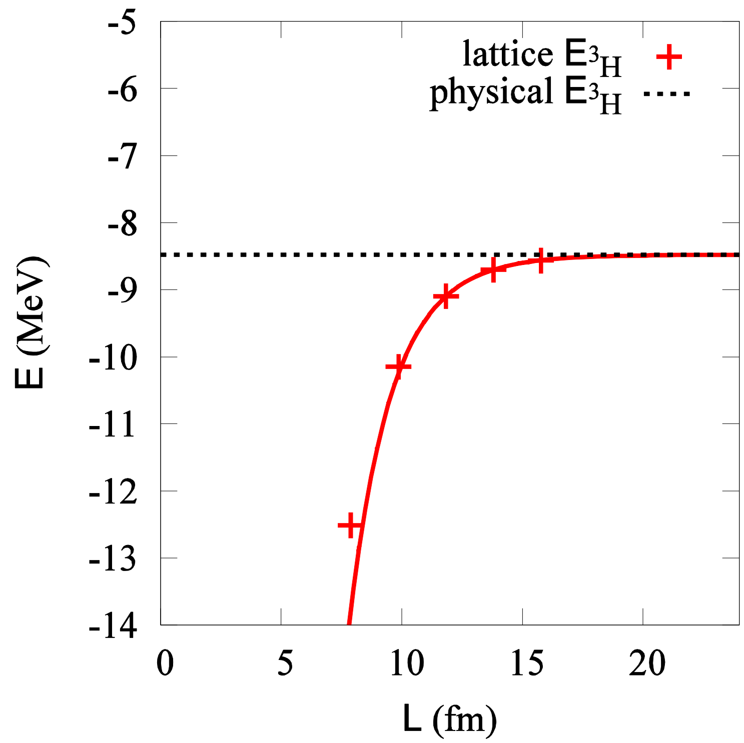


Three nucleon forces

Two unknown coefficients at NNLO from three-nucleon forces.
Determine c_D and c_E using ^{3}H binding energy and the weak axial current at low cutoff momentum.



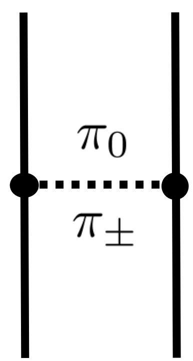
Park, et al., PRC 67 (2003) 055206,
Gårdestig, Phillips, PRL 96 (2006) 232301,
Gazit, Quaglioni, Navratil, PRL 103 (2009) 102502



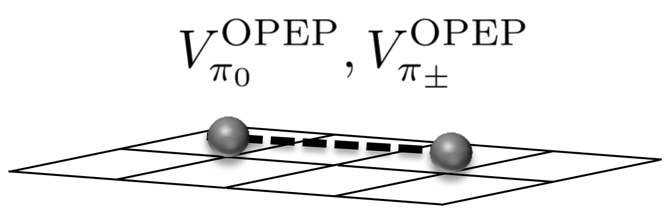
Neutrons and protons: Isospin breaking and Coulomb

Isospin-breaking and power counting *[Friar , van Kolck, PRC 60 (1999) 034006; Walzl, Meißner, Epelbaum NPA 693 (2001) 663; Friar, van Kolck, Payne, Coon, PRC 68 (2003) 024003; Epelbaum, Meißner, PRC72 (2005) 044001…]*

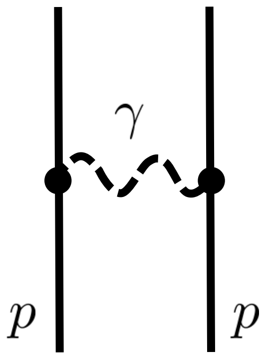
Pion mass difference

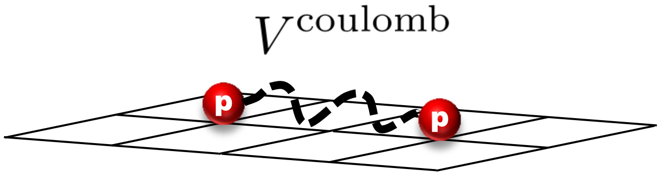


$m_{\pi_0} \neq m_{\pi_\pm}$

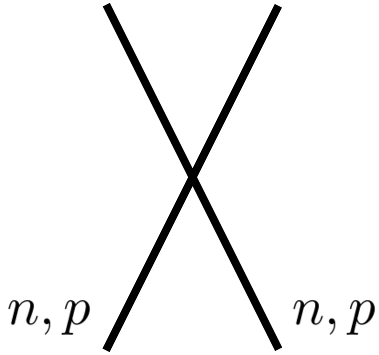


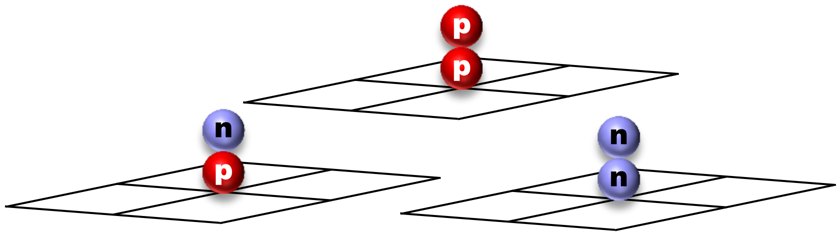
Coulomb potential





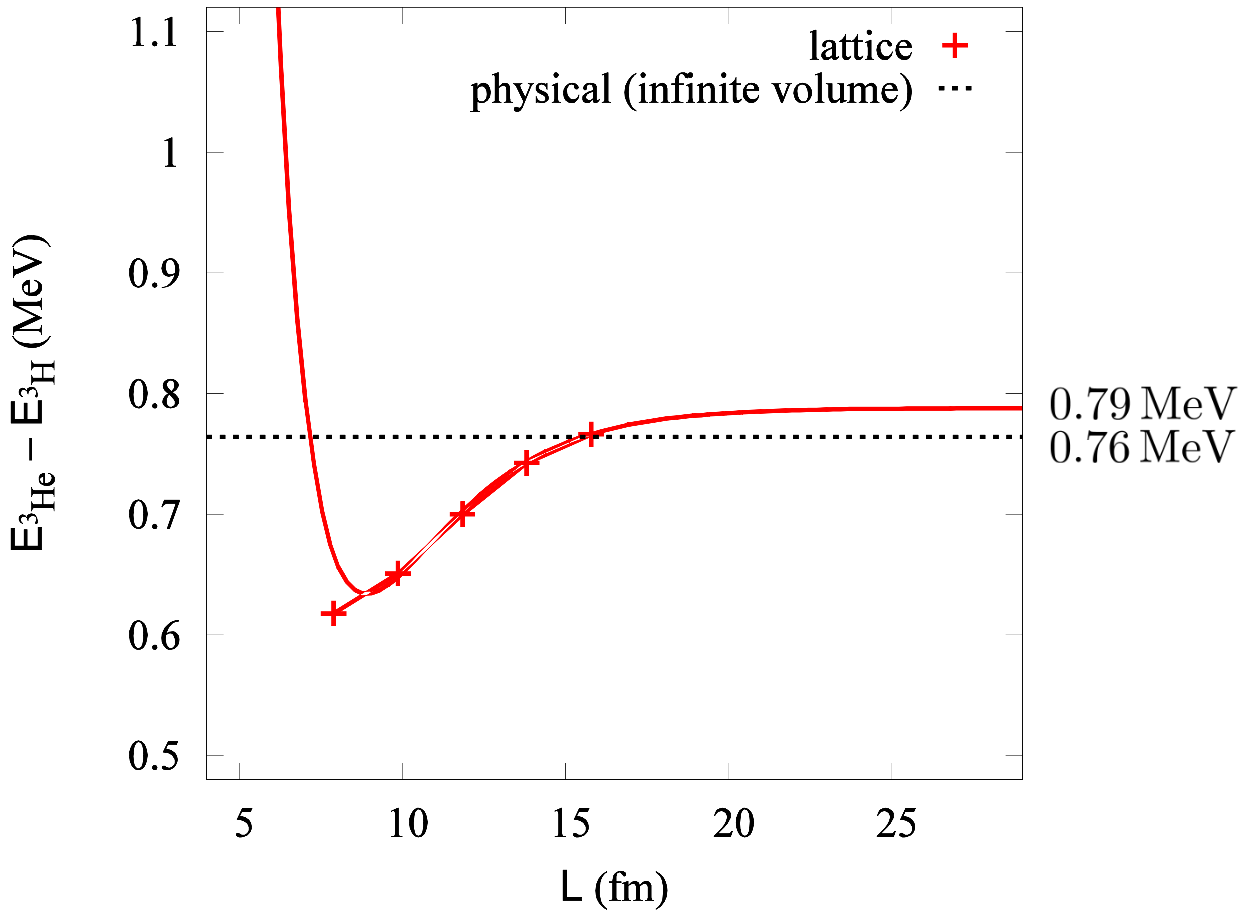
Charge symmetry breaking
Charge independence breaking



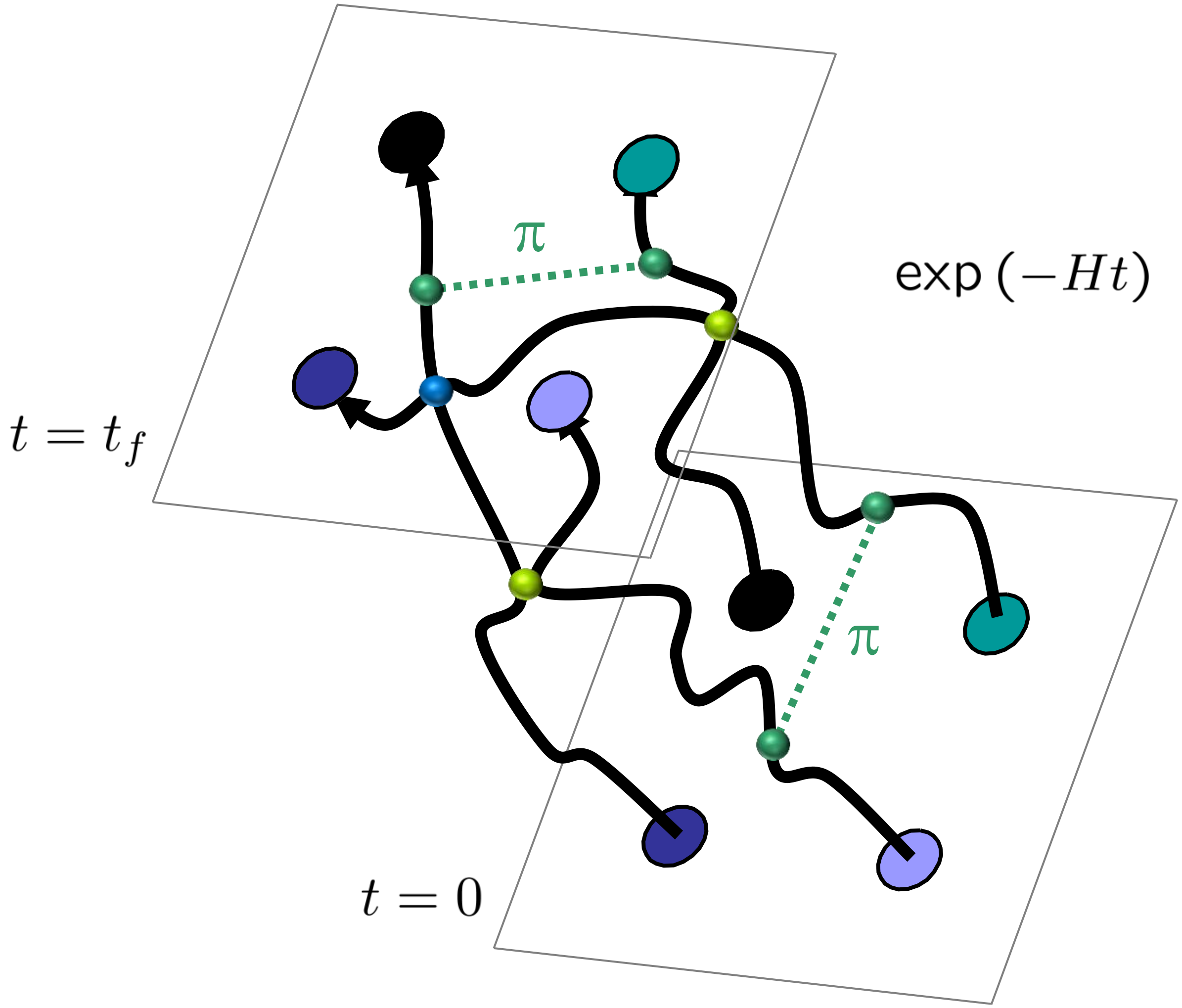


Triton and Helium-3

$$E_{^3\mathrm{He}} - E_{\mathrm{triton}} = 0.79(5)\ \mathrm{MeV}$$



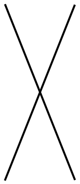
Euclidean time projection



Auxiliary field method

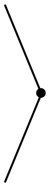
We can write exponentials of the interaction using a Gaussian integral identity

$$\exp\left[-\frac{C}{2}(N^{\dagger}N)^{2}\right]$$



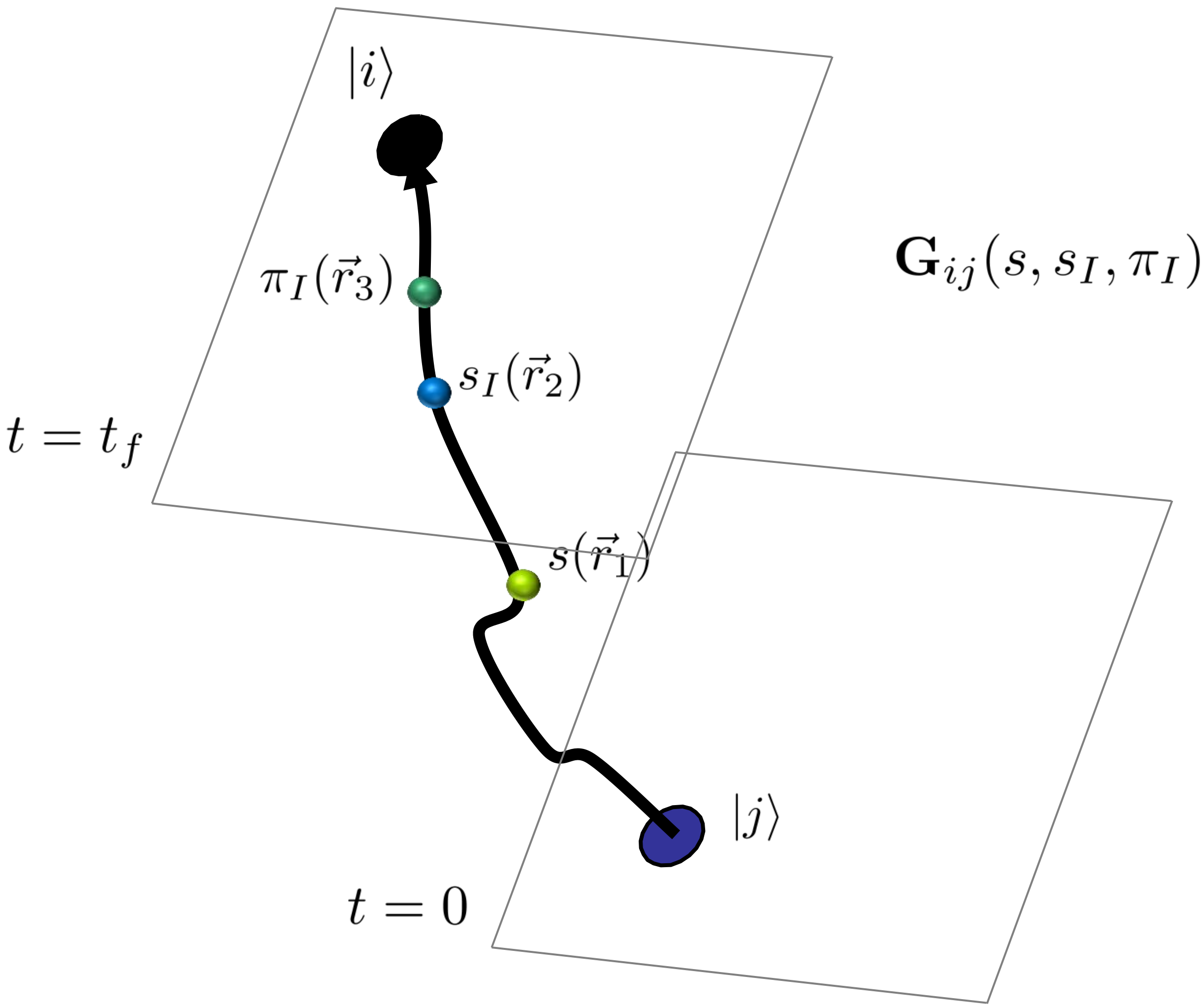
$(N^{\dagger}N)^{2}$

$$= \sqrt{\frac{1}{2\pi}}\int_{-\infty}^{\infty} ds \exp\left[-\frac{1}{2}s^{2} + \sqrt{-C}\, s(N^{\dagger}N)\right]$$

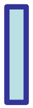


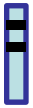
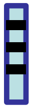
$sN^{\dagger}N$

We remove the interaction between nucleons and replace it with the interactions of each nucleon with a background field.

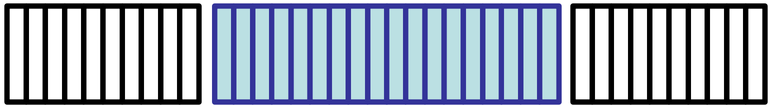

$|i\rangle$
$\pi_I(\vec{r}_3)$
$s_I(\vec{r}_2)$
$t = t_f$
$s(\vec{r}_1)$
$|j\rangle$
$t = 0$
$\mathbf{G}_{ij}(s, s_I, \pi_I)$

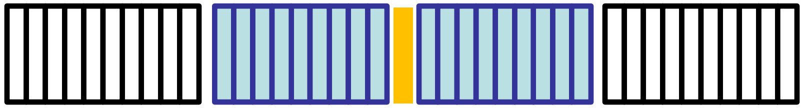
Schematic of lattice Monte Carlo calculation

 $= M_{\mathrm{LO}}$  $= M_{\mathrm{approx}}$  $= O_{\mathrm{observable}}$

 $= M_{\mathrm{NLO}}$  $= M_{\mathrm{NNLO}}$

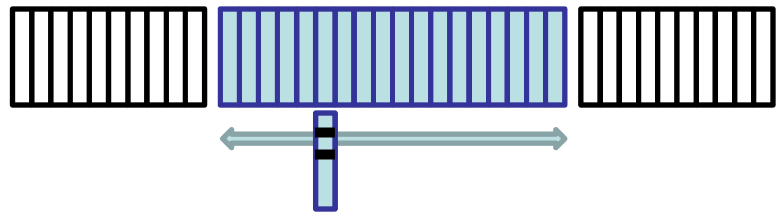
Hybrid Monte Carlo sampling

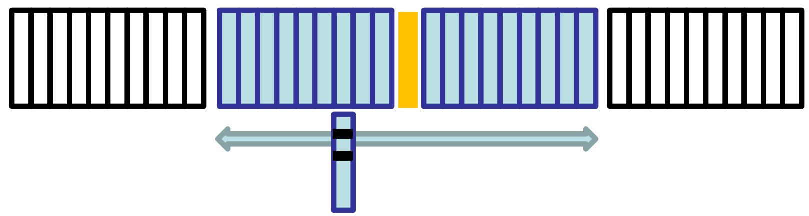
$Z_{n_t,\mathrm{LO}} = \langle \psi_{\mathrm{init}} |$  $| \psi_{\mathrm{init}} \rangle$

$Z^{\langle O \rangle}_{n_t,\mathrm{LO}} = \langle \psi_{\mathrm{init}} |$  $| \psi_{\mathrm{init}} \rangle$

$$e^{-E_{0,\mathrm{LO}} a_t} = \lim_{n_t \to \infty} Z_{n_t+1,\mathrm{LO}} / Z_{n_t,\mathrm{LO}}$$

$$\langle O \rangle_{0,\mathrm{LO}} = \lim_{n_t \to \infty} Z^{\langle O \rangle}_{n_t,\mathrm{LO}} / Z_{n_t,\mathrm{LO}}$$

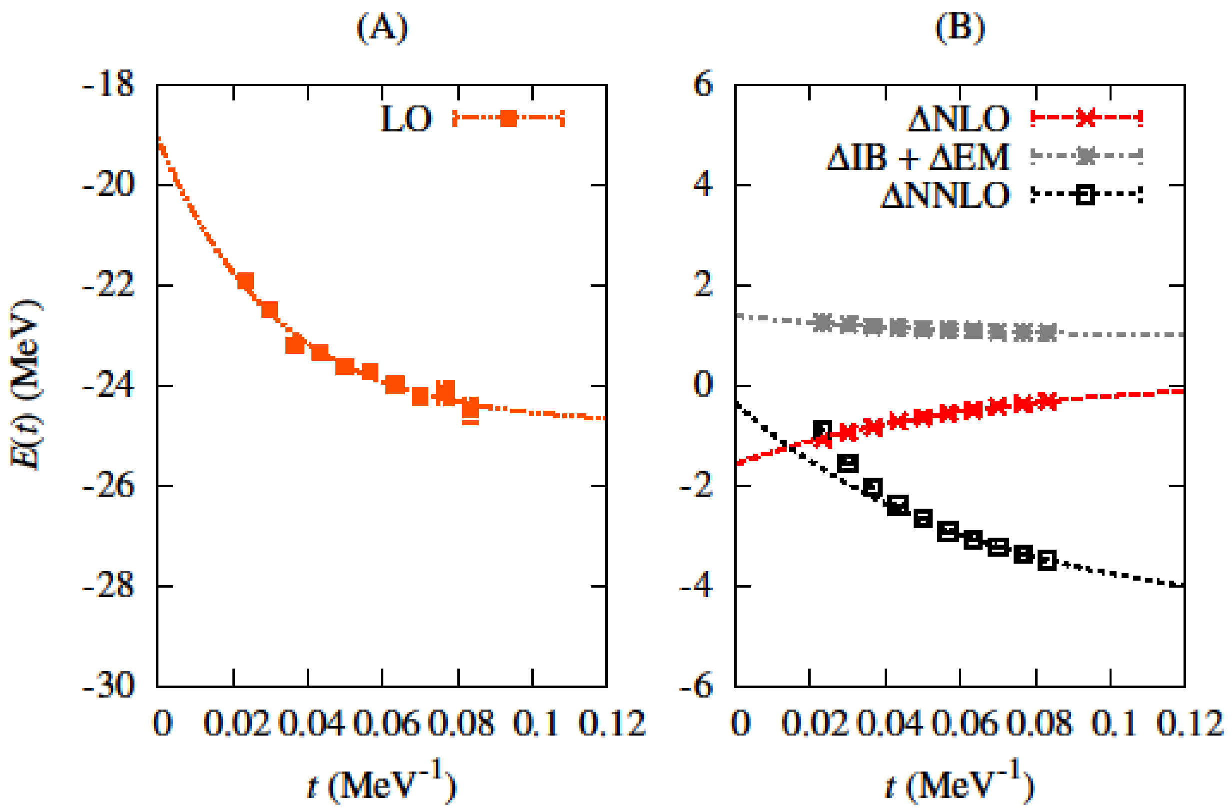
$Z_{n_t,\mathrm{NLO}} = \langle \psi_{\mathrm{init}} |$  $| \psi_{\mathrm{init}} \rangle$

$Z^{\langle O \rangle}_{n_t,\mathrm{NLO}} = \langle \psi_{\mathrm{init}} |$  $| \psi_{\mathrm{init}} \rangle$

$$\langle O \rangle_{0,\mathrm{NLO}} = \lim_{n_t \to \infty} Z^{\langle O \rangle}_{n_t,\mathrm{NLO}} / Z_{n_t,\mathrm{NLO}}$$

Ground state of Helium-4

$$L = 11.8\,\mathrm{fm}$$



Epelbaum, Krebs, D.L, Meißner, PRL 104 (2010) 142501; EPJA 45 (2010) 335; PRL 106 (2011) 192501

Ground state of Helium-4

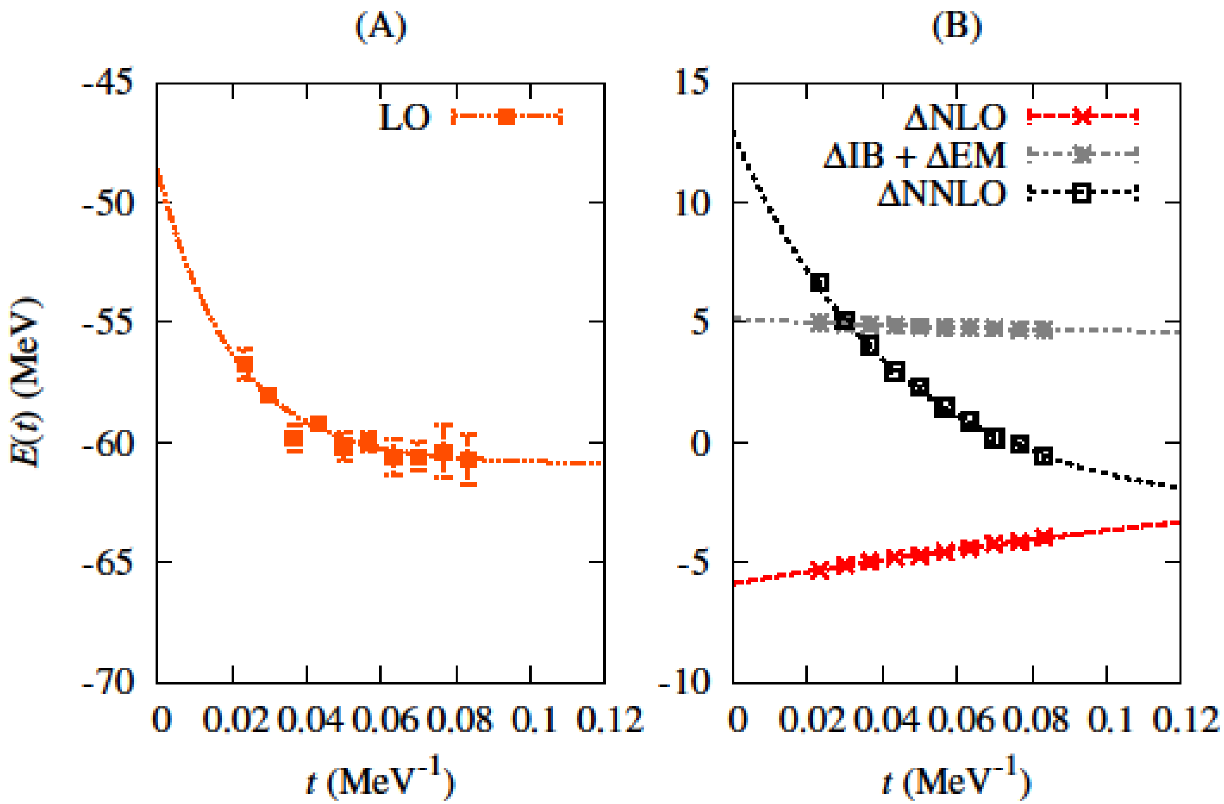
$$L = 11.8\,\mathrm{fm}$$

LO* ($O(Q^0)$)	–24.8(2) MeV
NLO ($O(Q^2)$)	–23.8(2) MeV
NNLO ($O(Q^3)$)	–28.4(3) MeV
Experiment	–28.3 MeV

*contains some interactions promoted from NLO

Ground state of Beryllium-8

$$L = 11.8\,\mathrm{fm}$$



Epelbaum, Krebs, D.L, Meißner, PRL 106 (2011) 192501

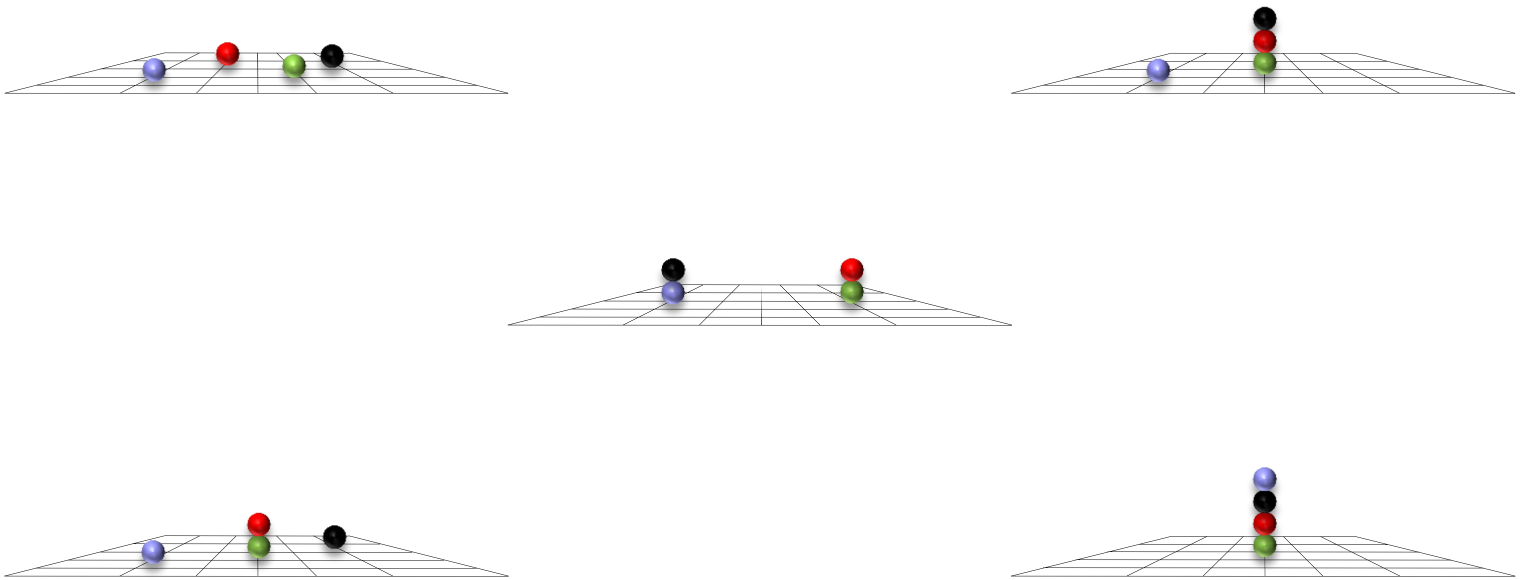
Ground state of Beryllium-8

$$L = 11.8\,\mathrm{fm}$$

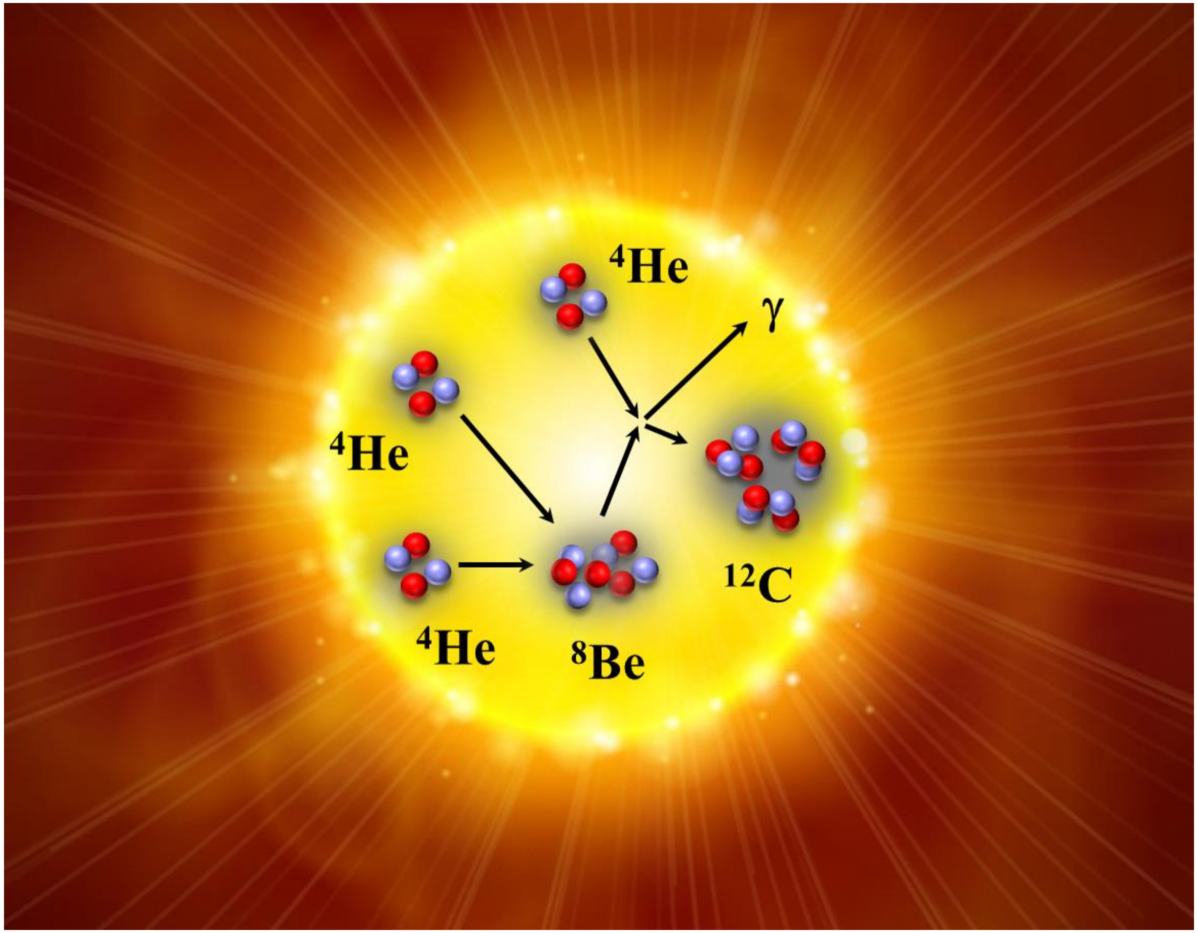
LO* ($O(Q^0)$)	–60.9(7) MeV
NLO ($O(Q^2)$)	–55(2) MeV
NNLO ($O(Q^3)$)	–58(2) MeV
Experiment	–56.5 MeV

*contains some interactions promoted from NLO

Particle clustering included automatically

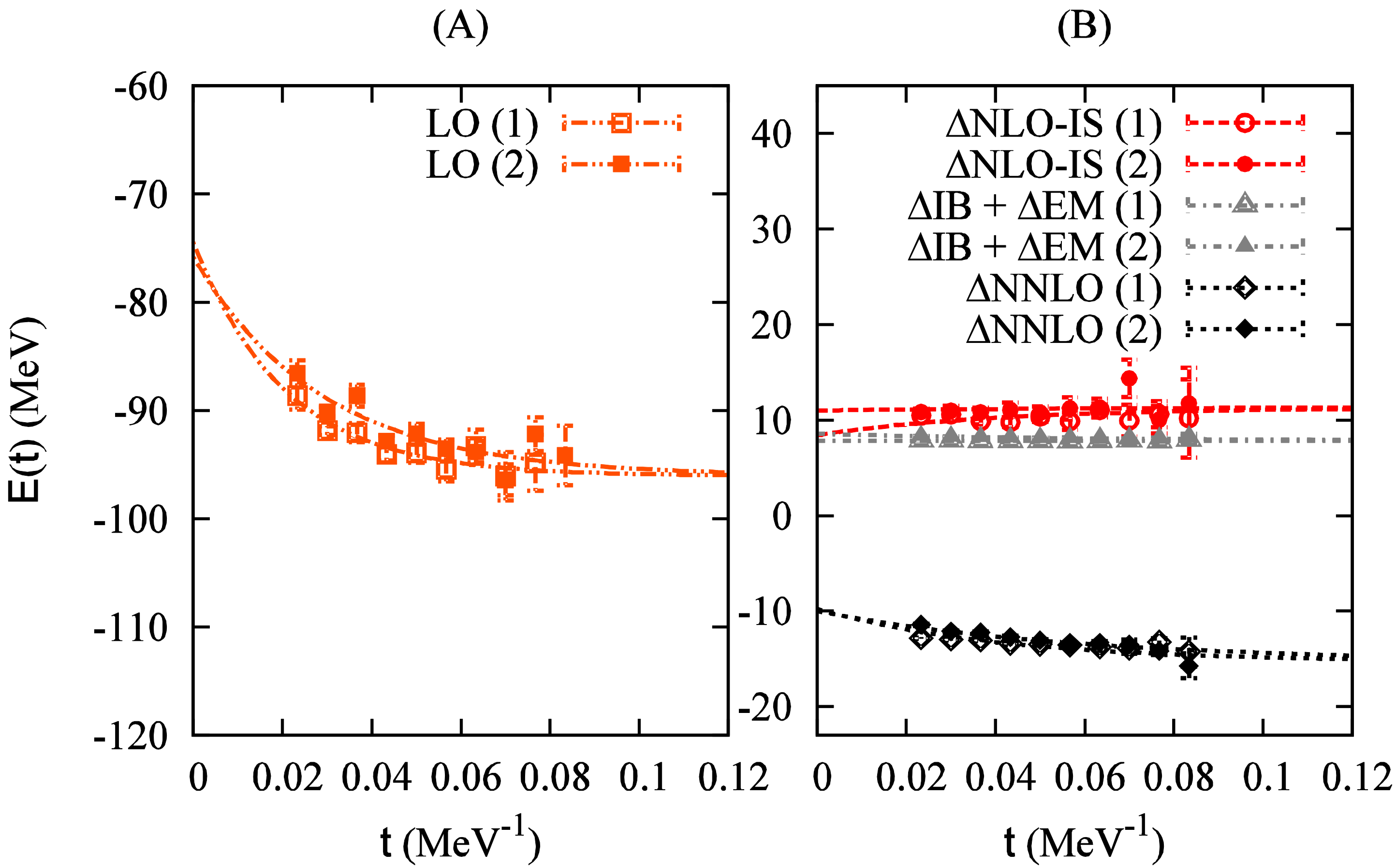


Carbon-12 spectrum and the Hoyle state



Ground state of Carbon-12

$$L = 11.8\,\mathrm{fm}$$



Epelbaum, Krebs, D.L, Meißner, PRL 106 (2011) 192501

Epelbaum, Krebs, Lähde, D.L, Meißner, PRL 109 (2012) 252501

Ground state of Carbon-12

$$L = 11.8\,\mathrm{fm}$$

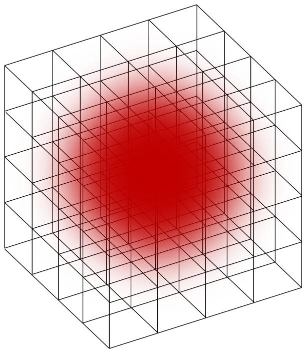
LO* ($O(Q^0)$)	–96(2) MeV
NLO ($O(Q^2)$)	–77(3) MeV
NNLO ($O(Q^3)$)	–92(3) MeV
Experiment	–92.2 MeV

*contains some interactions promoted from NLO

See also strong coupling lattice QCD results for carbon-12:

De Forcrand, Fromm, PRL 104 (2010) 112005

Simulations using general initial/final state wavefunctions



$$\bigwedge_{j=1,\cdots,A} |\psi_j(\vec{n})\rangle$$

Construct states with well-defined momentum using all possible translations.

$$L^{-3/2} \sum_{\vec{m}} e^{i\vec{P}\cdot\vec{m}} \bigwedge_{j=1,\cdots,A} |\psi_j(\vec{n}-\vec{m})\rangle$$

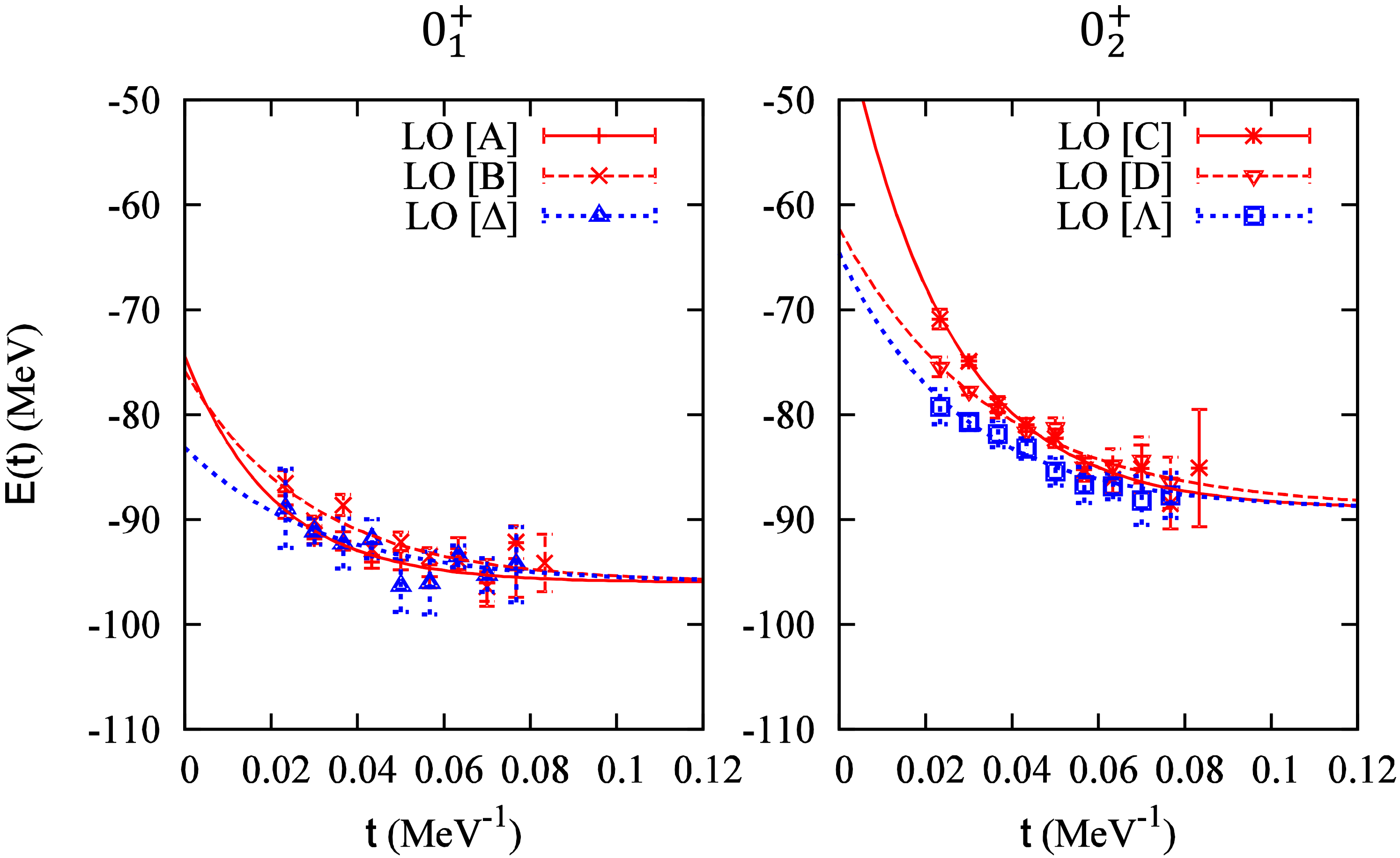
Shell model wavefunctions

$$\psi_j(\vec{n}) = \exp(-c\vec{n}^2)$$
$$\psi_j'(\vec{n}) = n_x \exp(-c\vec{n}^2)$$
$$\psi_j''(\vec{n}) = n_y \exp(-c\vec{n}^2)$$
$$\psi_j'''(\vec{n}) = n_z \exp(-c\vec{n}^2)$$
$$\vdots$$

Alpha cluster wavefunctions

$$\psi_j(\vec{n}) = \exp[-c(\vec{n} - \vec{m})^2]$$
$$\psi_j'(\vec{n}) = \exp[-c(\vec{n} - \vec{m}')^2]$$
$$\psi_j''(\vec{n}) = \exp[-c(\vec{n} - \vec{m}'')^2]$$
$$\vdots$$

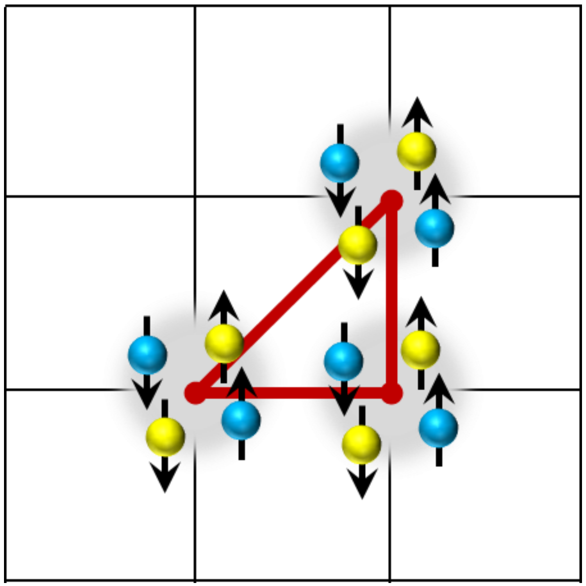
Carbon-12



Epelbaum, Krebs, Lähde, D.L, Meißner, PRL 109 252501 (2012)

Structure of ground state and first 2+

Strong overlap with compact triangle configuration

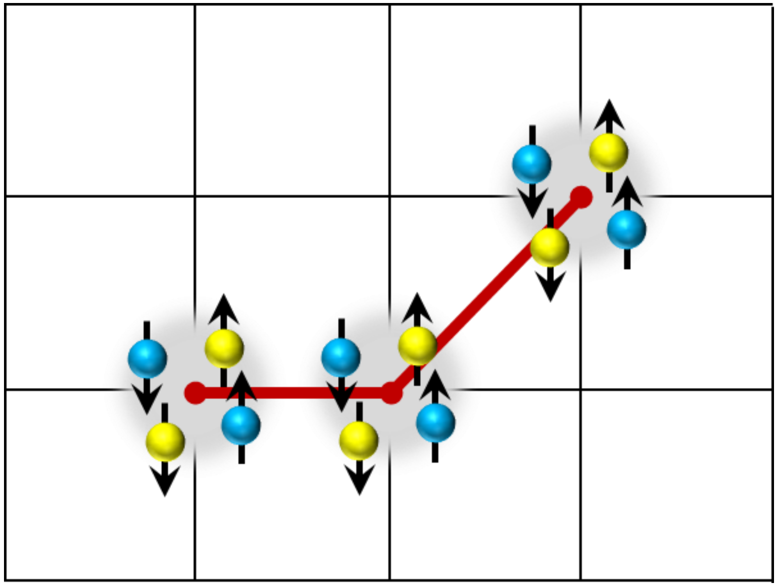


12 rotational orientations

$b = 1.97\ \mathrm{fm}$

Structure of Hoyle state and second 2+

Strong overlap with bent arm configuration



24 rotational orientations

$b = 1.97$ fm

Excited state spectrum of carbon-12 (even parity)

	2_1^+	0_2^+	2_2^+
LO* ($O(Q^0)$)	–94(2) MeV	–89(2) MeV	–88(2) MeV
NLO ($O(Q^2)$)	–74(3) MeV	–72(3) MeV	–70(3) MeV
NNLO ($O(Q^3)$)	–89(3) MeV	–85(3) MeV	–83(3) MeV
Experiment	–87.72 MeV	–84.51 MeV	–82.6(1) MeV (*A*,*B*) –81.1(3) MeV (*C*) –82.13(11) MeV (*D*)

*contains some interactions promoted from NLO

A – Freer et al., PRC 80 (2009) 041303

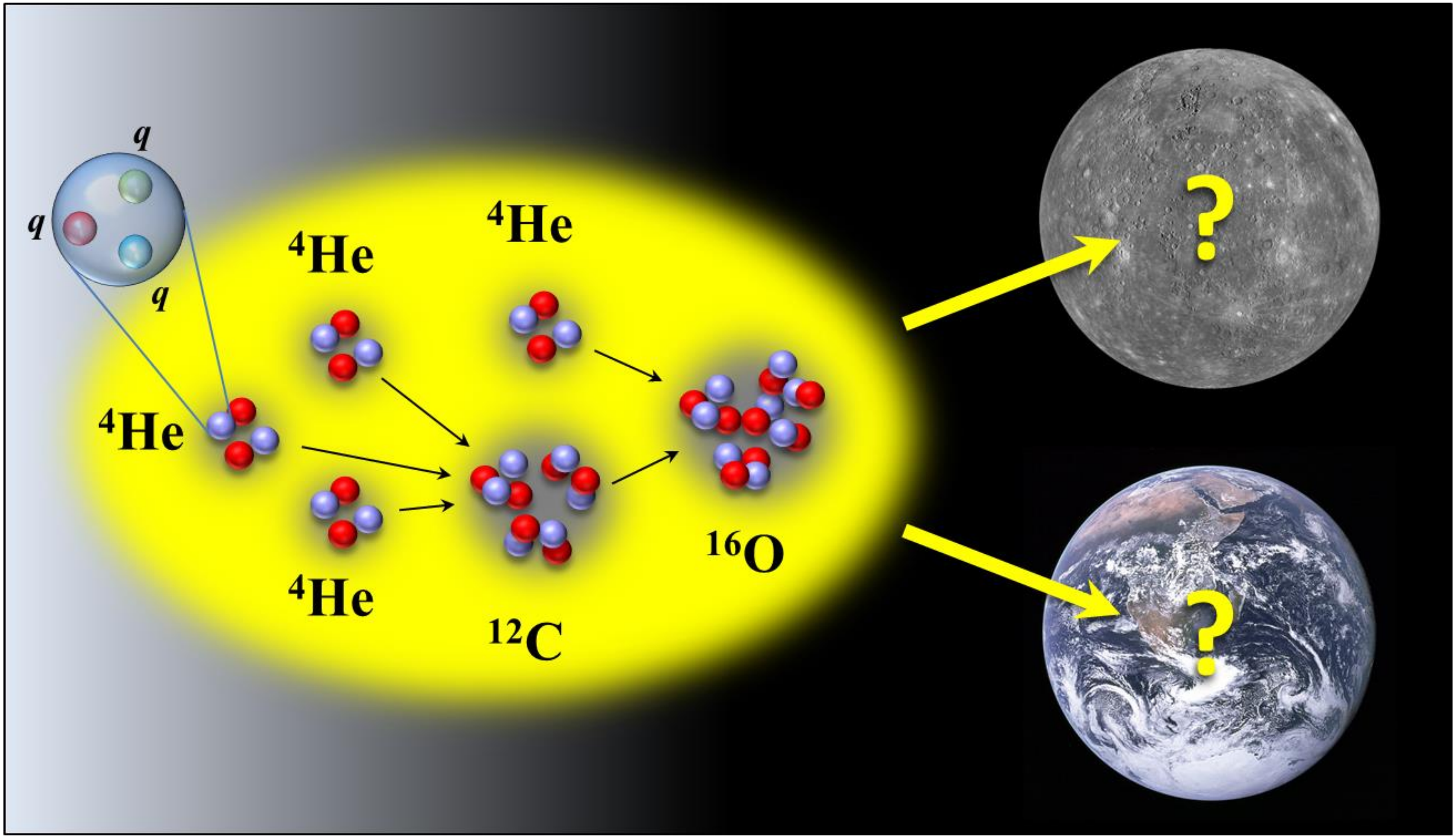
B – Zimmerman et al., PRC 84 (2011) 027304

C – Hyldegaard et al., PRC 81 (2010) 024303

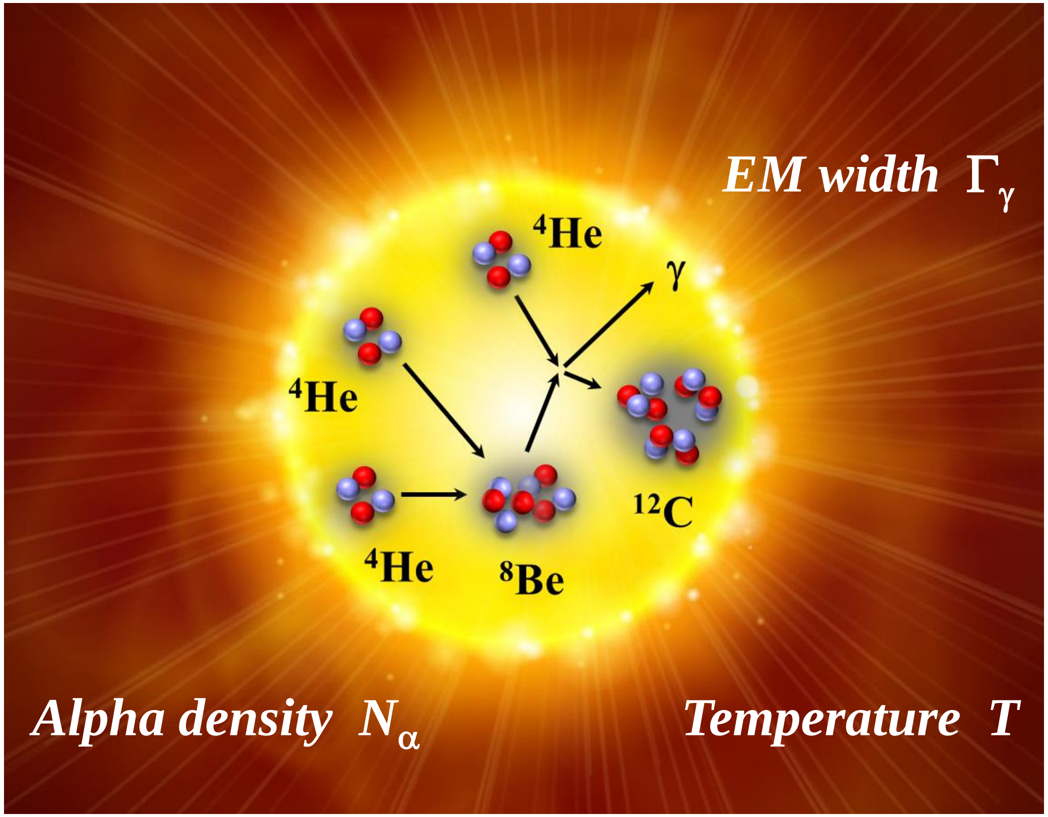
D – Itoh et al., PRC 84 (2011) 054308

Epelbaum, Krebs, Lähde, D.L, Meißner, PRL 109 252501 (2012)

Light quark mass dependence of helium burning



Triple alpha reaction rate



$$r_{3\alpha} \propto \Gamma_\gamma \left(N_\alpha / k_B T\right)^3 \times \exp(-\varepsilon / k_B T)$$

$\varepsilon = E_h - 3E_\alpha$ Hoyle relative to triple-alpha

Is nature fine-tuned?

$$\varepsilon = E_h - 3E_\alpha \approx 380\,\mathrm{keV}$$

$$\varepsilon > 480\,\mathrm{keV}$$

Less resonance enhancement. Rate of carbon production smaller by several orders of magnitude. Low carbon abundance is unfavorable for carbon-based life.

$$\varepsilon < 280\,\mathrm{keV}$$

Carbon production occurs at lower stellar temperatures and oxygen production greatly reduced. Low oxygen abundance is unfavorable for carbon-based life.

Schlattl et al., Astrophys. Space Sci., 291, 27–56 (2004)

We investigate the dependence on the fundamental parameters of the standard model such as the light quark masses. Can be parameterized by the pion mass.

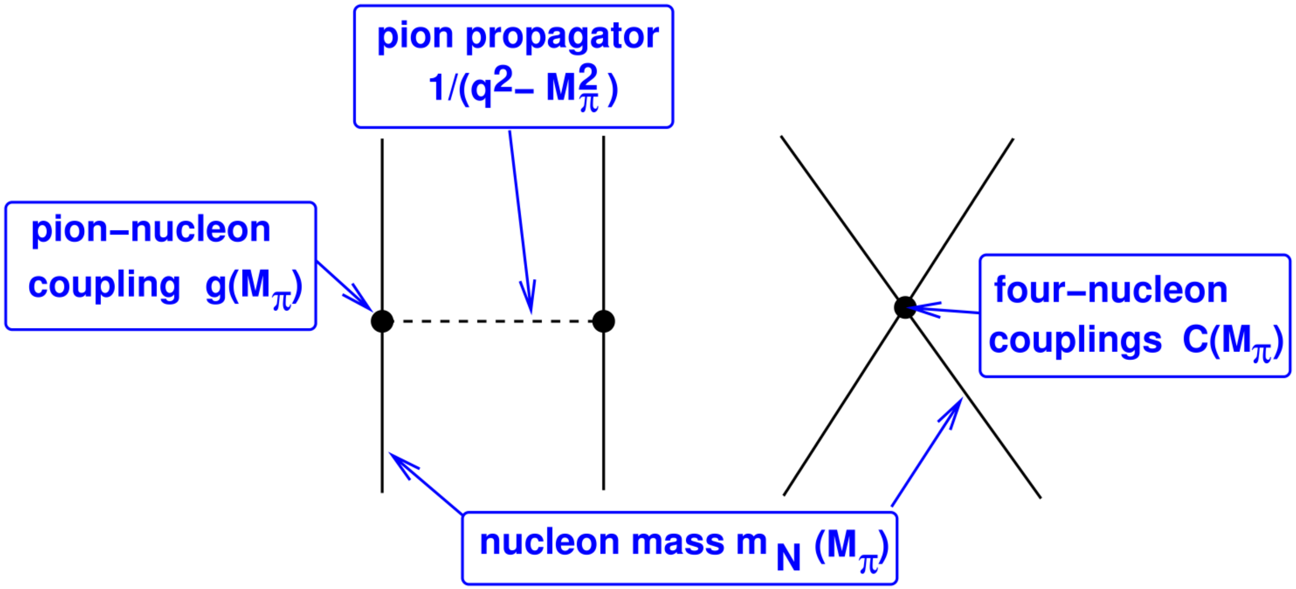
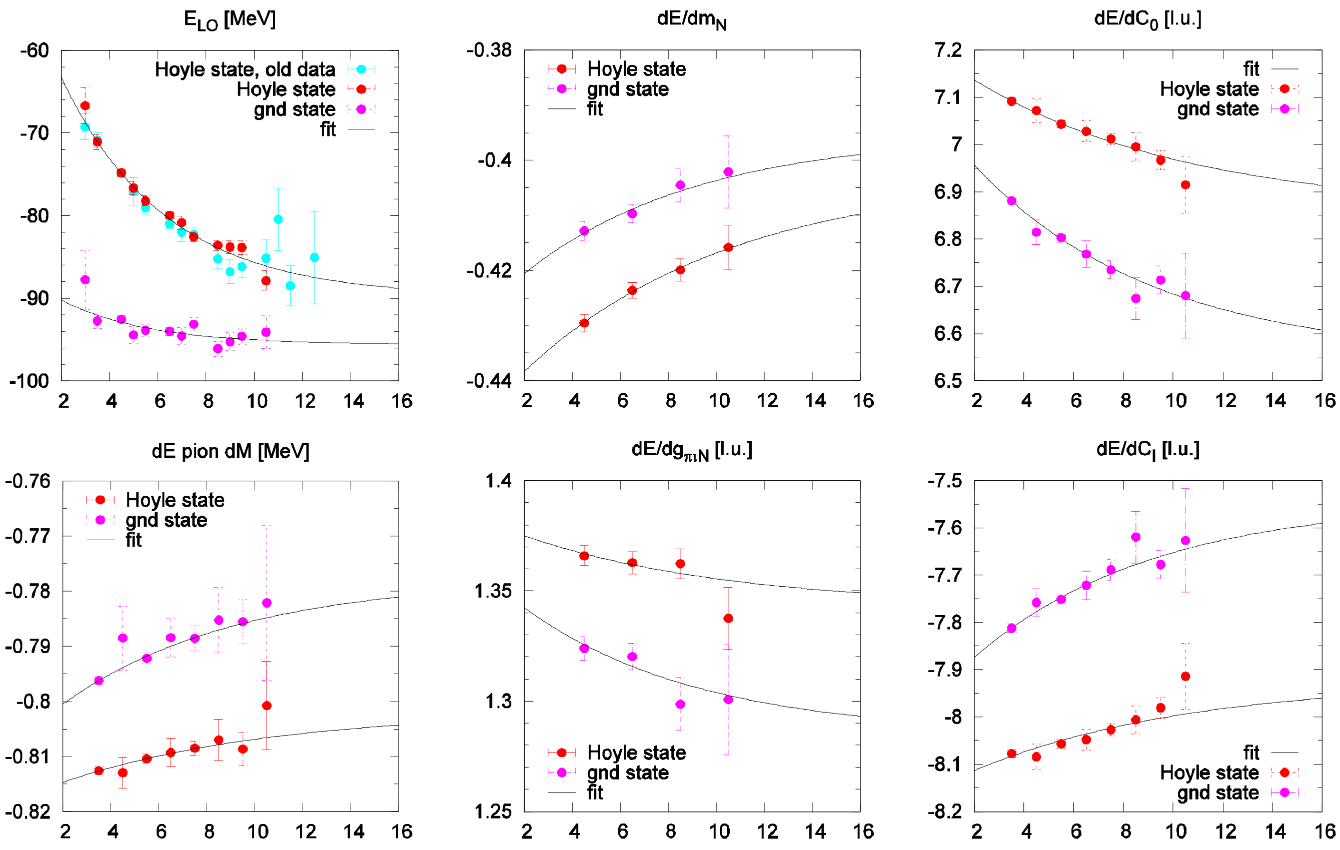


Figure courtesy of U.-G. Meißner

Epelbaum, Krebs, Lähde, D.L, Meißner, PRL 110 (2013) 112502; EJPA 49 (2013) 82; Berengut et al., Phys. Rev. D 87 (2013) 085018

Lattice results for pion mass dependence



$$\Delta E_h = E_h - E_b - E_\alpha$$ Hoyle relative to Be-8-alpha

$$\Delta E_b = E_b - 2E_\alpha$$ Be-8 relative to alpha-alpha

$$\varepsilon = E_h - 3E_\alpha$$ Hoyle relative to triple-alpha

$$\left.\frac{\partial \Delta E_h}{\partial M_\pi}\right|_{M_\pi^{\mathrm{ph}}} = -0.455(35)\bar{A}_s - 0.744(24)\bar{A}_t + 0.051(19)$$

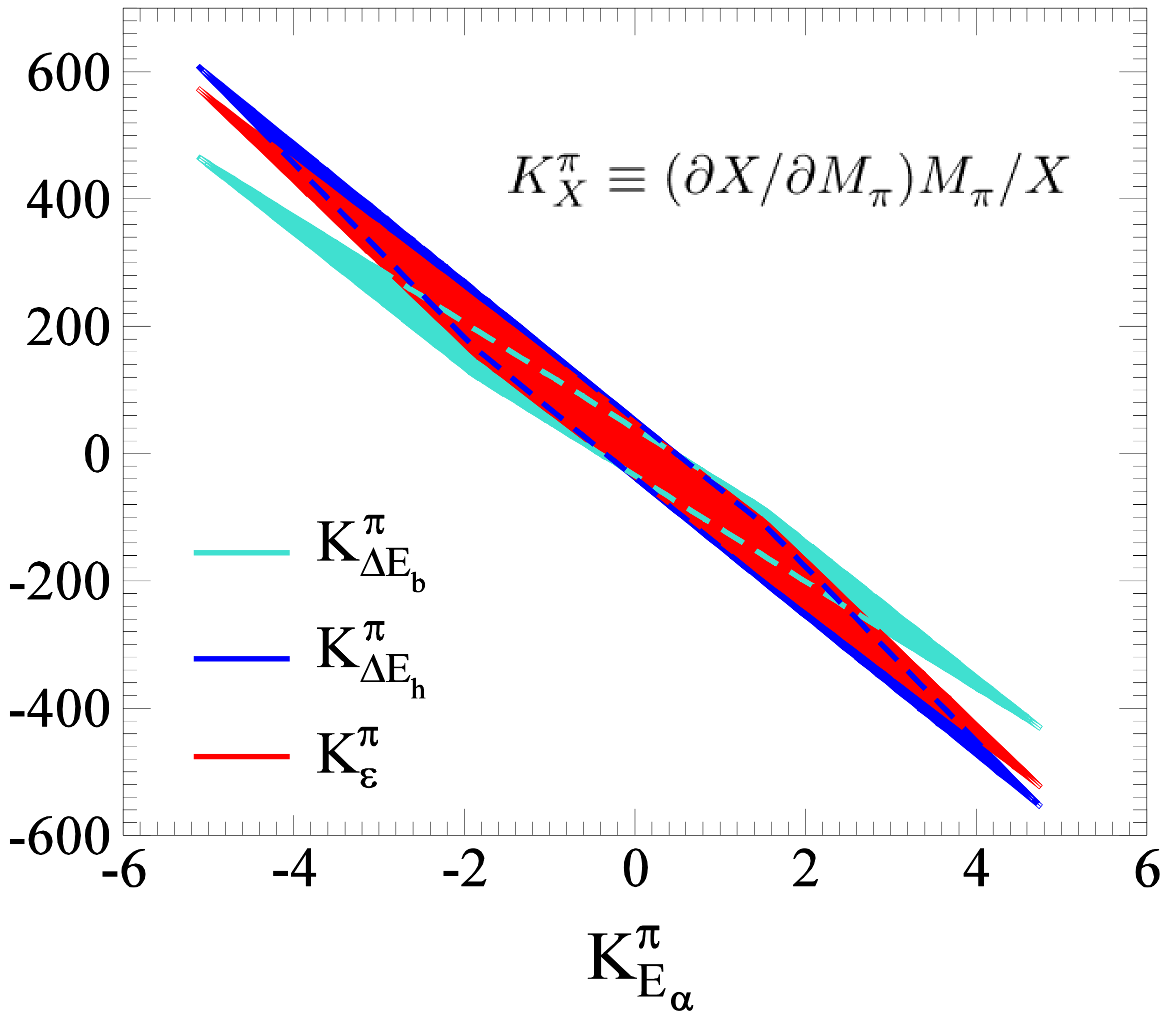
$$\left.\frac{\partial \Delta E_b}{\partial M_\pi}\right|_{M_\pi^{\mathrm{ph}}} = -0.117(34)\bar{A}_s - 0.189(24)\bar{A}_t + 0.013(12)$$

$$\left.\frac{\partial \varepsilon}{\partial M_\pi}\right|_{M_\pi^{\mathrm{ph}}} = -0.572(19)\bar{A}_s - 0.933(15)\bar{A}_t + 0.064(16)$$

$$\bar{A}_s \equiv \partial a_s^{-1}/\partial M_\pi\big|_{M_\pi^{\mathrm{ph}}} \qquad \bar{A}_t \equiv \partial a_t^{-1}/\partial M_\pi\big|_{M_\pi^{\mathrm{ph}}}$$

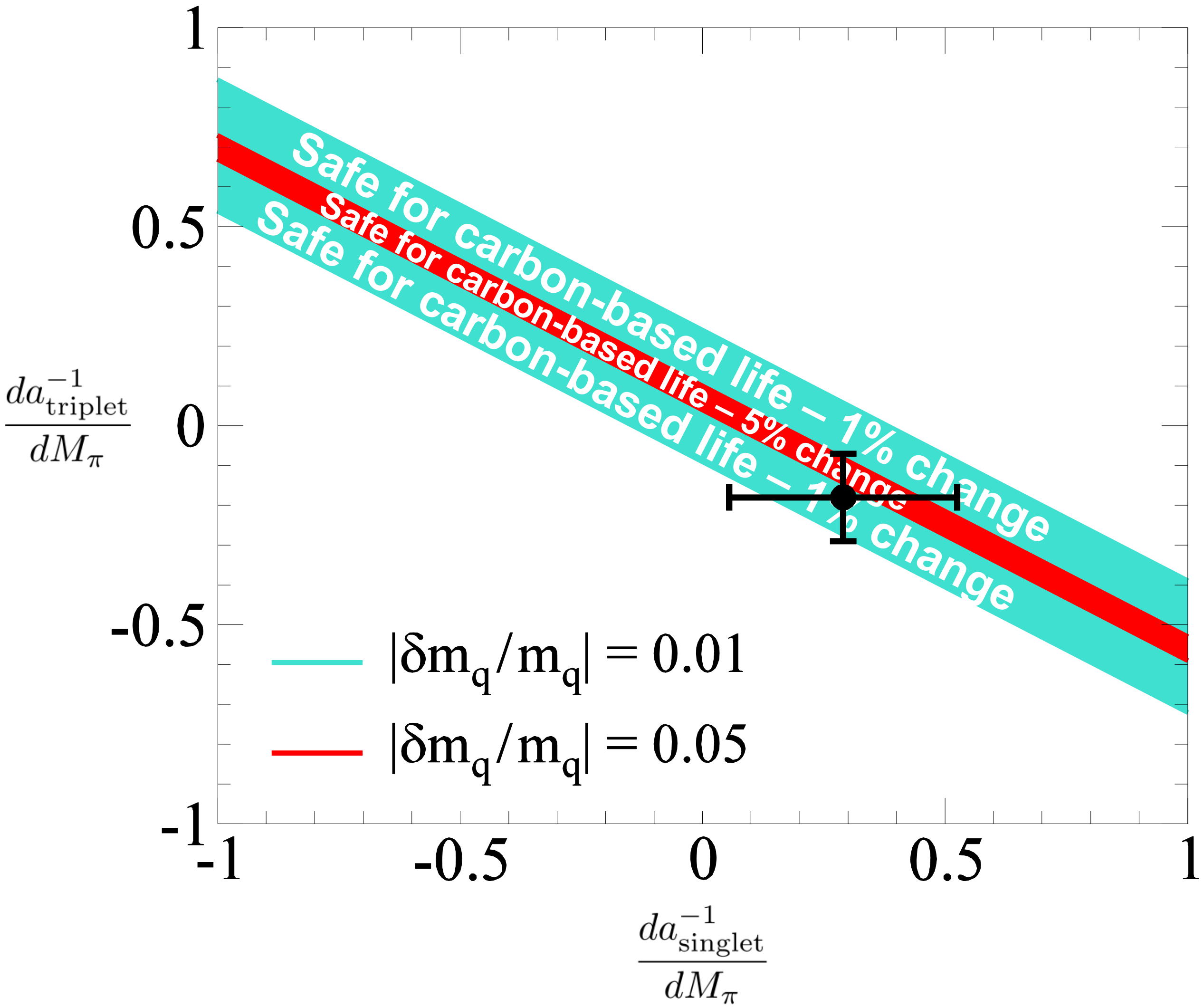
Epelbaum, Krebs, Lähde, D.L, Meißner, PRL 110 (2013) 112502; EJPA 49 (2013) 82; Berengut et al., Phys. Rev. D 87 (2013) 085018

Evidence for correlation with alpha binding energy



Epelbaum, Krebs, Lähde, D.L, Meißner, PRL 110 (2013) 112502; EJPA 49 (2013) 82

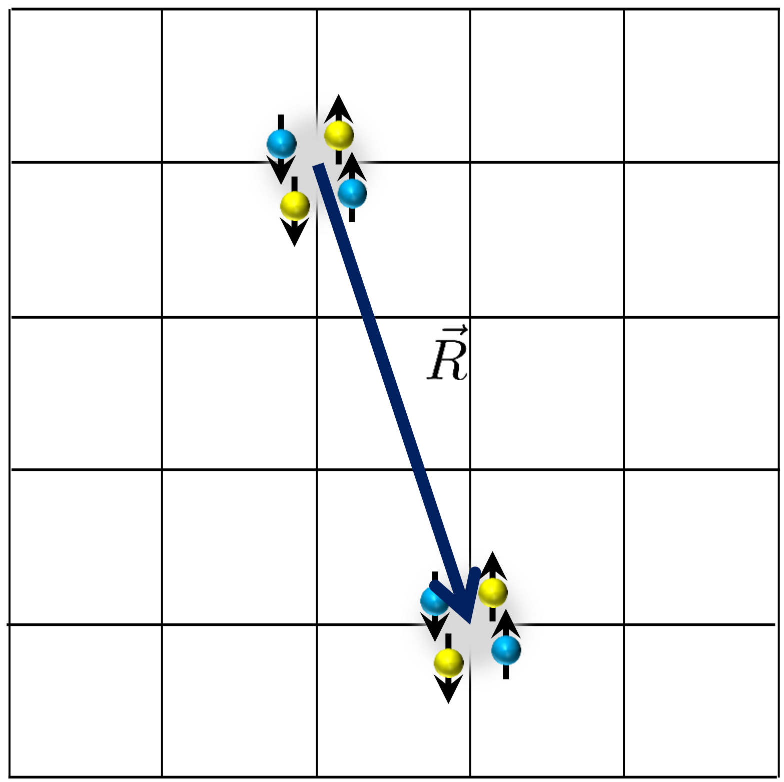
“End of the world” plot



Epelbaum, Krebs, Lähde, D.L, Meißner, PRL 110 (2013) 112502; EJPA 49 (2013) 82

Scattering and reactions on the lattice

Projected adiabatic matrix method

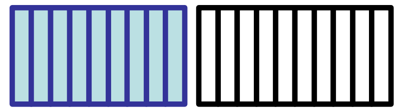


Using cluster wavefunctions for initial continuum scattering states

$$|\vec{R}>$$

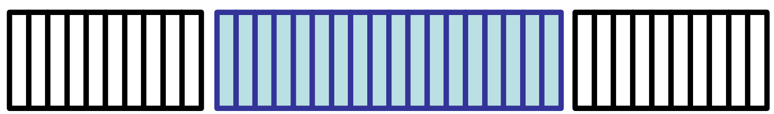
Use projection Monte Carlo to propagate cluster wavefunctions in Euclidean time

$$|\vec{R}>_t = e^{-Ht}|\vec{R}>$$

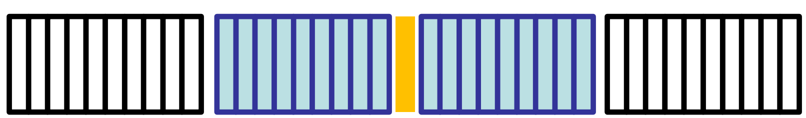
$|\vec{R}>_t =$  $|\vec{R}>$

Construct a norm matrix and matrix of expectation values

$$<N>_t = {}_t<\vec{R}'|\vec{R}>_t =$$

$<\vec{R}'|$  $|\vec{R}>$

$$<O>_t = {}_t<\vec{R}'|O|\vec{R}>_t =$$

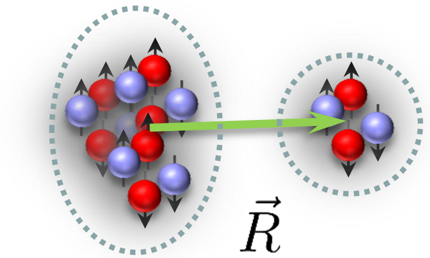
$<\vec{R}'|$  $|\vec{R}>$

Compute the projected adiabatic matrix

$$< O >_{\rm adiab} =< N >_t^{-1/2} < O >_t < N >_t^{-1/2}$$

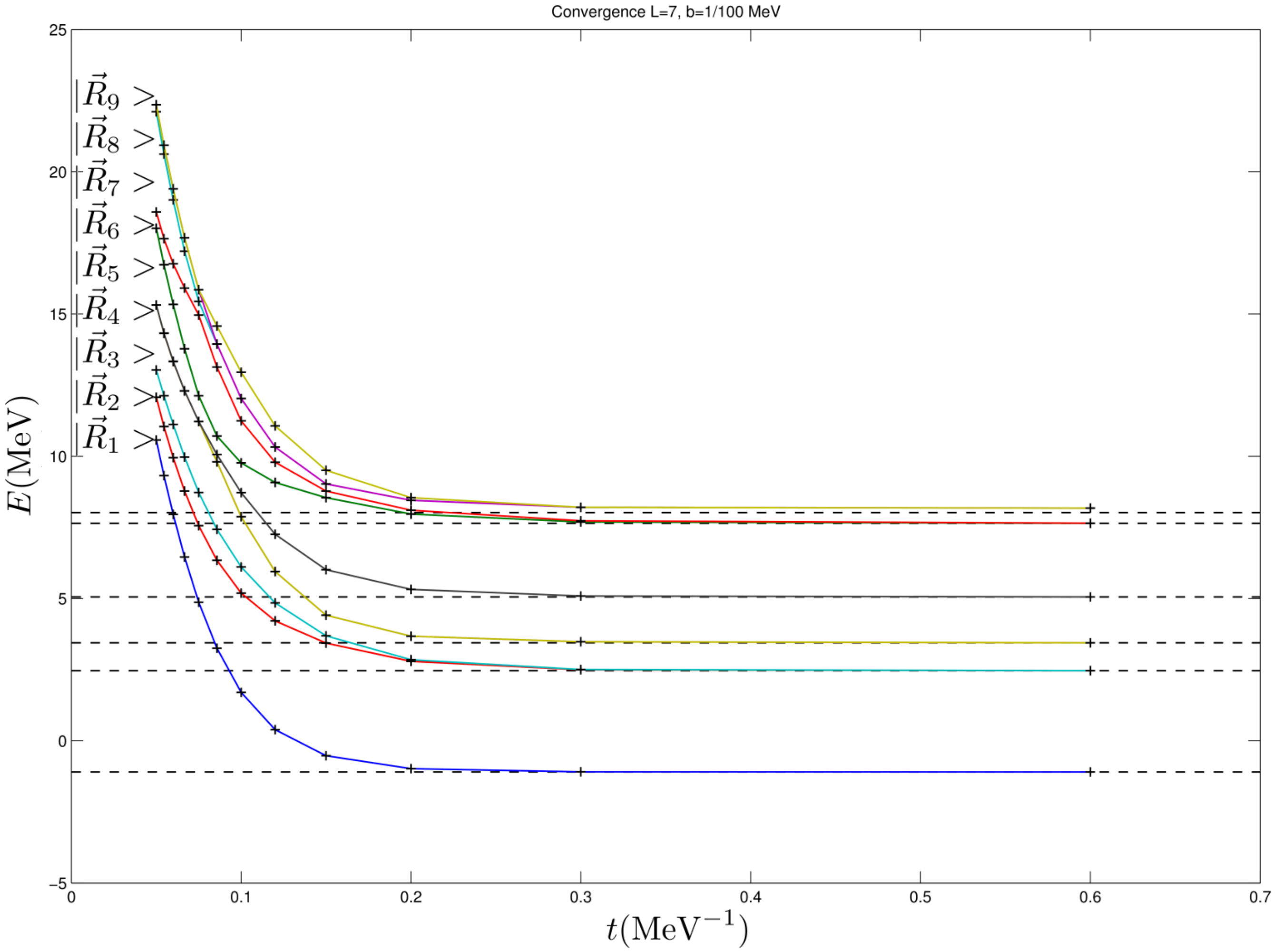
Projected adiabatic Hamiltonian is now an effective two-body Hamiltonian. Similar in spirit to no-core shell model with resonating group method.

But some differences. Distortion of the nucleus wavefunctions is automatic due to projection in Euclidean time.



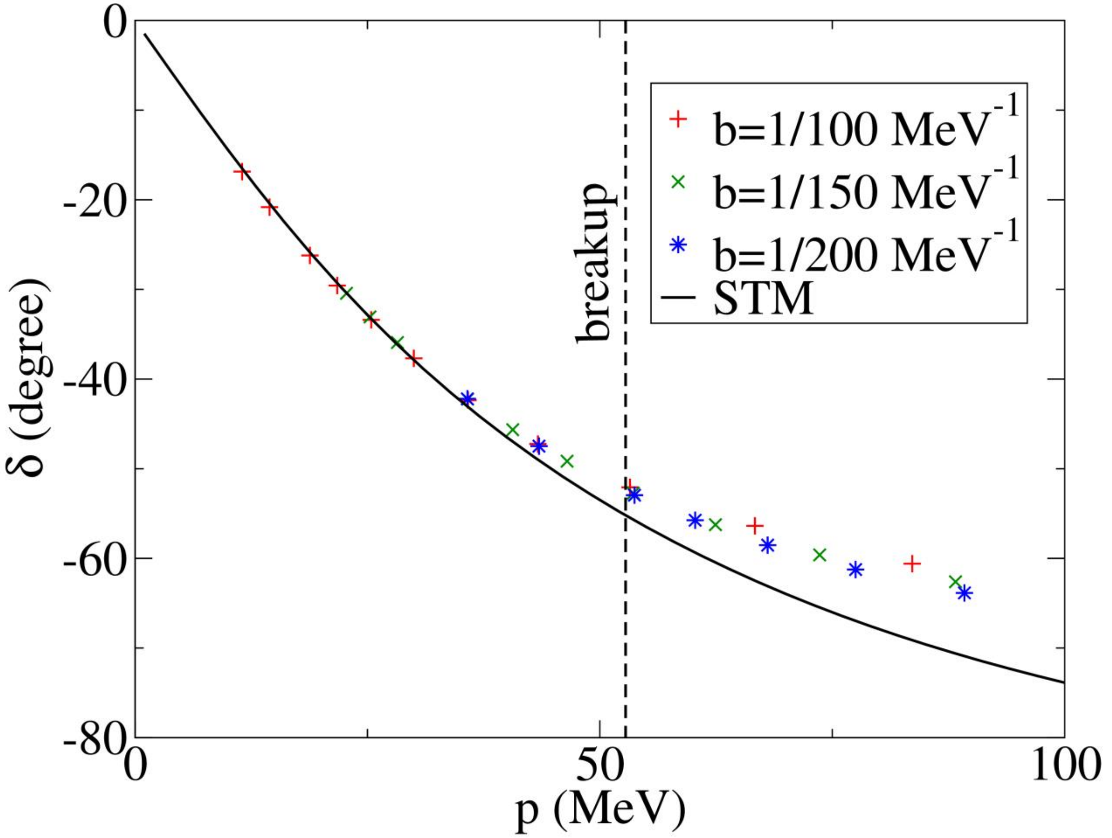
Rupak, D.L., PRL 111 (2013) 032502; Pine, D.L., Rupak, work in progress

Example: Quartet neutron-deuteron scattering



Pine, D.L., Rupak, work in progress

Quartet neutron-deuteron scattering (pionless EFT at LO)

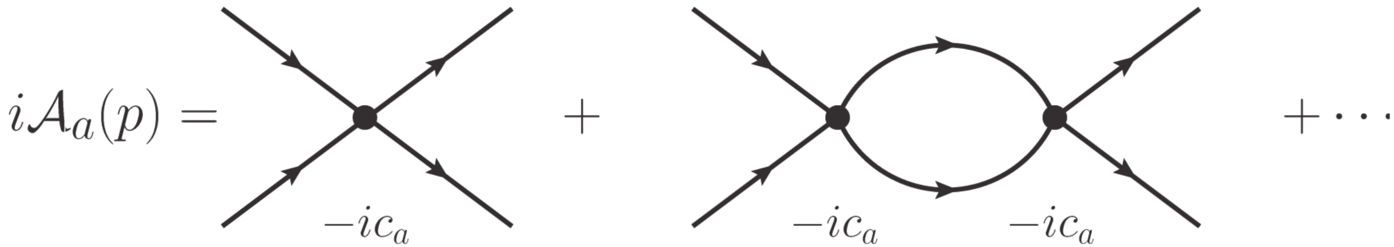


Pine, D.L., Rupak, work in progress

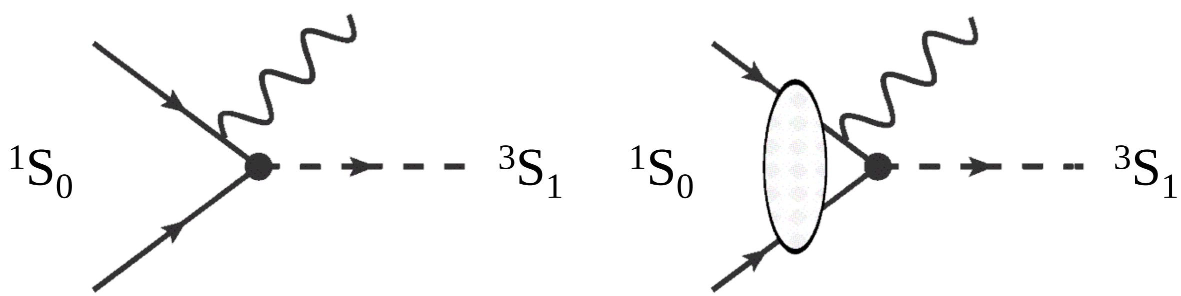
Use coupled channels for capture reactions and break up processes.

Lattice Green's function methods for radiative capture tested for $n + p \to d + \gamma$ in pionless effective field theory at leading order.

Elastic scattering amplitude ($^1S_0$ and $^3S_1$)

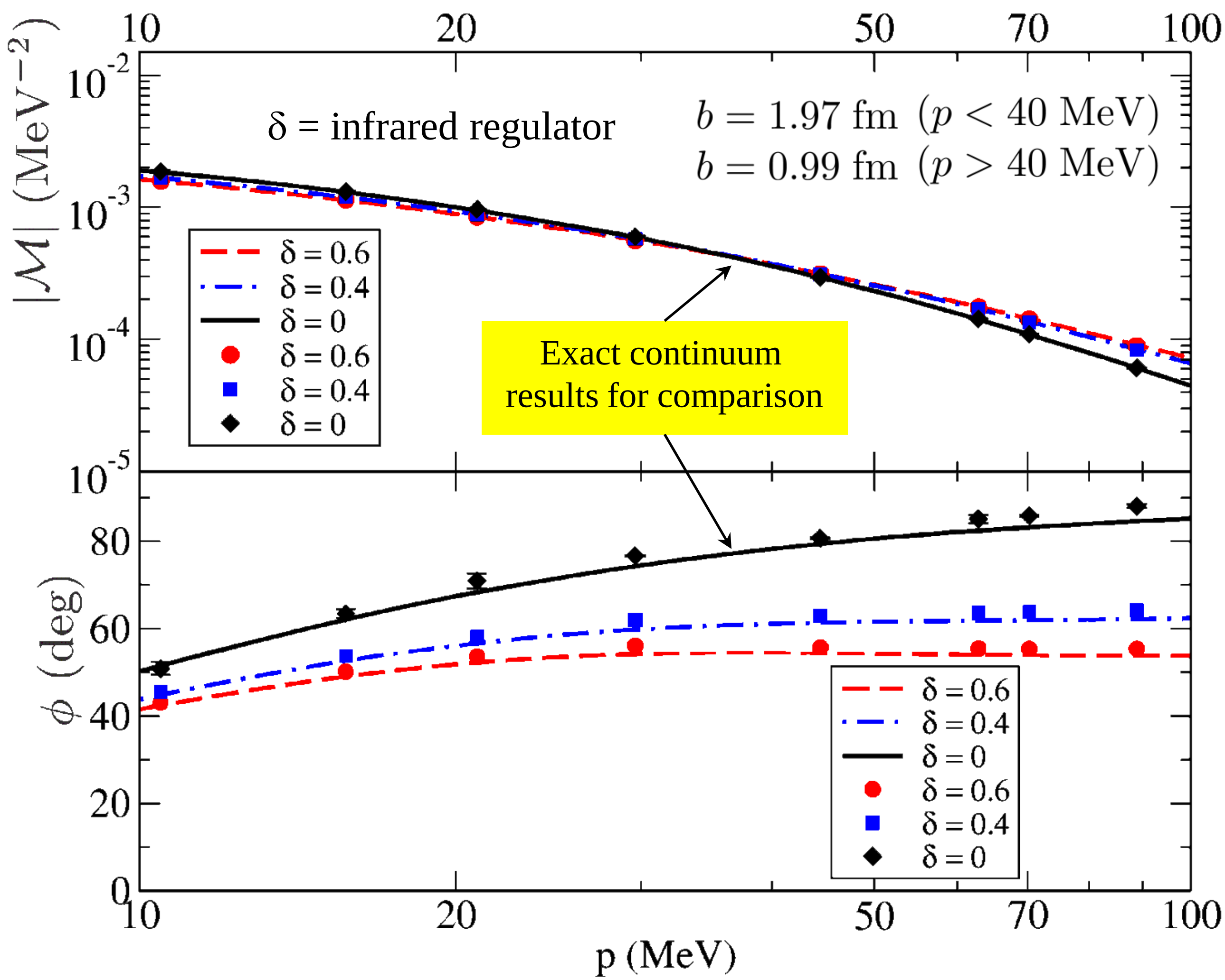


M1 radiative capture amplitude



Rupak, D.L., PRL 111 (2013) 032502

M1 transition amplitude $n + p \to d + \gamma$



Rupak, D.L., PRL 111 (2013) 032502

Summary

A golden age for nuclear theory from first principles. Big science discoveries being made and many more around the corner.

Lattice effective field theory is a relatively new and promising tool that combines the framework of effective field theory and computational lattice methods. May play a significant role in the future of *ab initio* nuclear theory.

Topics being addressed now and in the near future…

Different lattice spacings, $N \neq Z$ nuclei, neutron matter equation of state and superfluid transition from S-wave to P-wave, ground state nuclei up to $A = 28$, structure and spectrum of oxygen-16, etc.