QCD transition at finite temperature with Domain Wall fermions

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The symmetries of QCD

At the classical level, the symmetries of QCD with N_f flavors of massless fermions:

 $SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times U(1)_A$

Spontaneous SU(N_f)_LxSU(N_f)_R chiral symmetry breaking

gives rise to 8 Goldstone bosons: the π, K, η

9th Goldstone boson η' ?

• $U(I)_A$ symmetry is violated by axial anomaly at the quantum level and is responsible for the η - η ' mass splitting 't Hooft, Adler, Bell & Jackiw, Witten & Veneziano

$$\partial_{\mu} j_{5}^{\mu} = \frac{g^{2} N_{f}}{16\pi^{2}} tr(\tilde{F}_{\mu\nu} F^{\mu\nu}), \qquad m_{\eta}^{2} + m_{\eta'}^{2} - 2m_{K}^{2} = \frac{2N_{f}}{F_{\pi}^{2}} \chi_{top}^{N_{f}=0}$$

fate of $U(I)_A$ symmetry at finite T and its consequences



First principle calculations on the lattice

Difficulties:

• chiral fermions that preserve exact chiral symmetry and produce correct axial anomaly are needed

Ginsparg-Wilson relation: ${D^{-1}, \gamma_5}=a\gamma_5$

 Overlap fermions: the only operator satisfies the Ginsparg-Wilson relation, however, there exists a "freezing" topology problem, more expensive than Domain Wall fermions

• Domain Wall fermions: preserve exact chiral symmetry and produce correct axial anomaly when the fifth dimension is sufficiently large. Residual symmetry breaking is quantified by the additive renormalization factor m_{res} to the quark mass

Current work:

simulations using Domain Wall fermions with two pion masses: m_{π} =200 MeV and 135 MeV on Nt=8 lattices on various volumes

results are shown for m_{π} =200 MeV if not mentioned explicitly

signatures of chiral symmetry restoration

• Susceptibilities defined as integrated correlation functions of the eight local operators, e.g. $\chi_{\pi} = \int d^4x < \pi^i(x)\pi^i(0) > 1$



• Dirac Eigenvalue spectrum $\rho(\lambda)$ • P(0)=0 signals the restoration of SU(2)_LxSU(2)_R Banks-Casher relation: $\langle \overline{\Psi}\Psi \rangle = \pi\rho(0)$ • A sizable gap from zero in $\rho(\lambda)$ signals U(1)_A restoration

$$\chi_{\pi} - \chi_{\delta} = \int_0^\infty d\lambda \,\rho(\lambda, \tilde{m}) \,\left(\frac{2\tilde{m}}{\tilde{m}^2 + \lambda^2}\right)^2$$

signatures of chiral symmetry restorations



The connection to Dirac eigenvalue spectrum

$$\langle \bar{\psi}\psi \rangle = \int_0^\infty d\lambda \,\rho(\lambda,\tilde{m}) \,\frac{2\tilde{m}}{\tilde{m}^2 + \lambda^2} + \frac{\langle |Q_{\rm top}| \rangle}{\tilde{m}V}$$
$$\chi_\pi - \chi_\delta = \int_0^\infty d\lambda \,\rho(\lambda,\tilde{m}) \,\left(\frac{2\tilde{m}}{\tilde{m}^2 + \lambda^2}\right)^2 + \frac{2\langle |Q_{\rm top}| \rangle}{\tilde{m}^2V}$$

• The first formula gives the Banks-Casher relation $\langle \overline{\Psi}\Psi \rangle = \pi \rho(0)$ in the chiral and infinite volume limit

The second terms are from exact zero mode contributions



The mild volume dependence of chiral condensates and differences of π and δ at T \gtrsim Tc indicates negligible exact zero mode contributions

exact DWF ward Identity

Gell-Mann-Oakes-Renner relation: $\langle \bar{\psi}\psi \rangle = m \, \chi_{\pi}$

In DWF formalism:

$$\langle \bar{\psi}\psi \rangle_l = (m_l + m_{res})\chi_{\pi_l} + R_{5d}^l$$

$$= m_l\chi_{\pi_l} + \Delta_{mp}^l$$

$$\langle \bar{\psi}\psi \rangle_s = (m_s + m_{res})\chi_{\pi_s} + R_{5d}^s$$

$$= m_s\chi_{\pi_s} + \Delta_{mp}^s$$



• subtracted pbp $\Delta_{I,s}$ to cancel the linear UV divergence in quark mass

$$\Delta_{l,s} = \langle \bar{\psi}\psi \rangle_l - \frac{m_l + m_{res}}{m_s + m_{res}} \langle \bar{\psi}\psi \rangle_s \quad \Delta_{l,s} = \Delta_{l,s}^{imp} + R_{5d}^l - \frac{m_l + m_{res}}{m_s + m_{res}} R_{5d}^s$$

• improved subtracted pbp $\Delta_{l,s}^{imp}$: suitable to DWF, cancel further residual chiral symmetry breaking effects, useful to be compared with results using e.g. Staggered fermions

$$\Delta_{l,s}^{\rm imp} = (m_l + m_{res})(\chi_{\pi_l} - \chi_{\pi_s})$$

improved subtracted chiral condensate



• the upward spikes with otherwise flat structure seen from the time history of chiral condensates comes from isolated, near zero modes

• Subtracted chiral condensates can be reproduced well from Dirac Eigenvalue spectrum $\rho(\lambda)$

• Features of Dirac Eigenvalue spectrum $\rho(\lambda)$ are accurate even in the low energy region where near zero modes locate

Dirac Eigenvalue spectrum with m_{π} =200 MeV



red histograms: 32^3 results $T < T_c$ nonzero $\rho(0)$ $T \sim T_c$ vanishing $\rho(0)$

black lines: 16³ results

T>T_c no gap from zero

see also in Sayantan Sharma's Poster

Z. Lin, PoS LATTICE2012 (2012) 084

relative importance of different contributions



parameterization of Dirac Eigenvalue spectrum

$$\rho(\tilde{m},\lambda) = a_0 + a_1 \,\tilde{m}^2 \,\delta(\lambda) + a_2 \,\lambda$$
 $\chi_{\pi} - \chi_{\delta} = \pi a_0 / \tilde{m} + 2a_1 + 2a_2$

a0: $SU(2)_L x SU(2)_R$ symmetry breaking contribution a1: near zero modes contribution

a2: new, $U(I)_A$ symmetry breaking, linear infrared behavior

 $SU(2)_L x SU(2)_R$ symmetry breaking term dominates below T_c while near zero modes contribution dominates above T_c

Underlying mechanism of $U(I)_A$ breaking

black lines: results from 16³ lattices, red histograms: results from 32³ lattices





of near zero modes per configuration

T [MeV]	16 ³ x8	32 ³ x8
177	0.15(2)	I.6(2)
186	0.013(6)	0.47(7)
195	0.056(9)	0.32(6)

 Resulting from non-zero global topology number of exact zero modes ~ √V density of exact zero modes ~ I/√V
 In a relatively dilute gas of instantons and anti-instantons (DIGA) number of near zero modes ~ V density of near zero modes independent of V

No evidence of $\rho(\lambda)$ shrinking by a factor of sqrt(8) from 16³ to 32³ lattices is found, which favors DIGA

Chirality of near zero modes $\chi_n = \frac{\int d^4 x \overline{\Psi}_n(x,0)(1+\gamma^5)\Psi_n(x,0) - \int d^4 x \overline{\Psi}_n(x,L_s-1)(1-\gamma^5)\Psi(x,L_s-1)}{\int d^4 x \overline{\Psi}_n(x,0)(1+\gamma^5)\Psi(x,0) + \int d^4 x \overline{\Psi}_n(x,L_s-1)(1-\gamma^5)\Psi(x,L_s-1)}$ $16^3 \times 8 - \widetilde{m}_l$ 0.003177 MeV N_0 : total # of # of configurations with N_0 and N_+ near zero $\rho(\Lambda) (\text{GeV}^3)$ 0.002 modes 32³x8,T=177 MeV N₊:# of near 0.001 N_+ | 0 1 2 3 4 5 zero modes $N_0 = 1 | 28 \ 19 \ - \ - \$ with positive 0 0.025 0.03 0.035 0.005 0.01 0.04 0.015 $N_0 = 2 | 16 | 19 | 12 - -$ chirality $\Lambda (\text{GeV})$ Z. Lin, PoS LATTICE2012 (2012) 084 $N_0 = 3 | 4 | 11 | 8 | 3 -$ zero modes resulting from non-zero global topology $N_0 = 4 \begin{vmatrix} 1 & 3 & 4 & 3 & 0 \end{vmatrix}$ distribution of chirality obeys bimodal

distribution of chirality obeys binomial data behaviors more like a binomial contribution

dilute instanton gas model

a dilute instanton gas model can describe the non-zero $U(I)_A$ breaking above T_c !

 $N_0 = 5 \mid 0 \quad 2 \quad 1 \quad 1 \quad 1 \quad 0$

mass dependence of chiral symmetry restorations



O(N) scaling behavior in the high temperature region



According to O(N) scaling the chiral susceptibility in the high temperature region is independent on h

The mass independence of chiral susceptibility observed in the high temperature region supports the breaking of U(1)_A symmetry

Summary

• We study the chiral observables/Dirac Eigenvalue spectrum on Nt=8 lattices with Domain Wall Fermions using two pion masses: m_{π} =200 MeV and 135 MeV

• An Exact DWF ward identity is observed

• The U(I)_A symmetry remains broken up to $1.2 T_{\chi SB}$

• The dilute instanton gas model provides a good description of the U(1)_A breaking above T_c