GOETHE UNIVERSITÄT FRANKFURT AM MAIN

XQCD 2013 - Bern, Switzerland

Gauge Corrections to the Phase Diagram of Strong Coupling LQCD

Wolfgang Unger, Owe Philipsen (University Frankfurt), Philippe de Forcrand (ETHZ, CERN), Jens Langelage (ETHZ)

HIC for FAIR Helmholtz International Center

Strong Coupling QCD - Motivation and Setup

Why Strong Coupling QCD?

- SC-LQCD exhibits **confinement** and **chiral symmetry breaking**.
- SC-LQCD phase diagram: study nuclear phase transition, possible for arbitrarily large chemical potential: the **sign problem** is **mild** (discrete time) or even absent (continuous time).
- Send the **gauge coupling to infinity:** $g \to \infty \quad \Rightarrow \quad \beta = \frac{2N_c}{g^2} \to 0$.
- Allows to integrate out the gauge fields completely! However, lattice remains coarse.

SC-LQCD with staggered fermions:

- First integrate out gauge fields analytically, as the link integration factorizes, then integrate out fermions.
- New degrees of freedom (exact rewriting of QCD path integral [1]):
  - **Monomers** correspond to mesons, $M(x) = \bar{\chi}(x)\chi(x)$,
  - **Dimers** correspond to meson hoppings (non-oriented),
  - **Baryons** form self-avoiding oriented loops, $B(x) = \frac{1}{N_c}\epsilon_{i_1 \dots i_{N_c}}\chi_{i_1}(x)\dots\chi_{i_{N_c}}(x)$.
- Strong coupling partition function after Grassmann integrals carried out (leading to the constraint):

$$\mathcal{Z}(m_q,\mu,\gamma) = \sum_{\{k,n,\ell\}} \underbrace{\prod_{b=(x,\mu)} \frac{(N_c-k_b)!}{N_c! k_b!}\gamma^{2k_b\delta_{\mu 0}}}_{\text{meson hoppings } M_x M_y} \underbrace{\prod_x \frac{N_c!}{n_x!}(2am_q)^{n_x}}_{\text{chiral condensate } M_x} \underbrace{\prod_\ell w(\ell,\mu)}_{\text{baryon hoppings } \bar{B}_x B_y} \qquad \sum_{\hat{\mu}} k_{\hat{\mu}} + n_x = N_c$$

The Phase Diagram in the Strong Coupling Limit

- behavior at low $a\mu$ qualitatively the same, first order transition strongly N_τ-dependent
- N_τ-dependence of phase boundary due to anisotropy γ, no re-entrance in continuous time ($N_\tau \to \infty$)

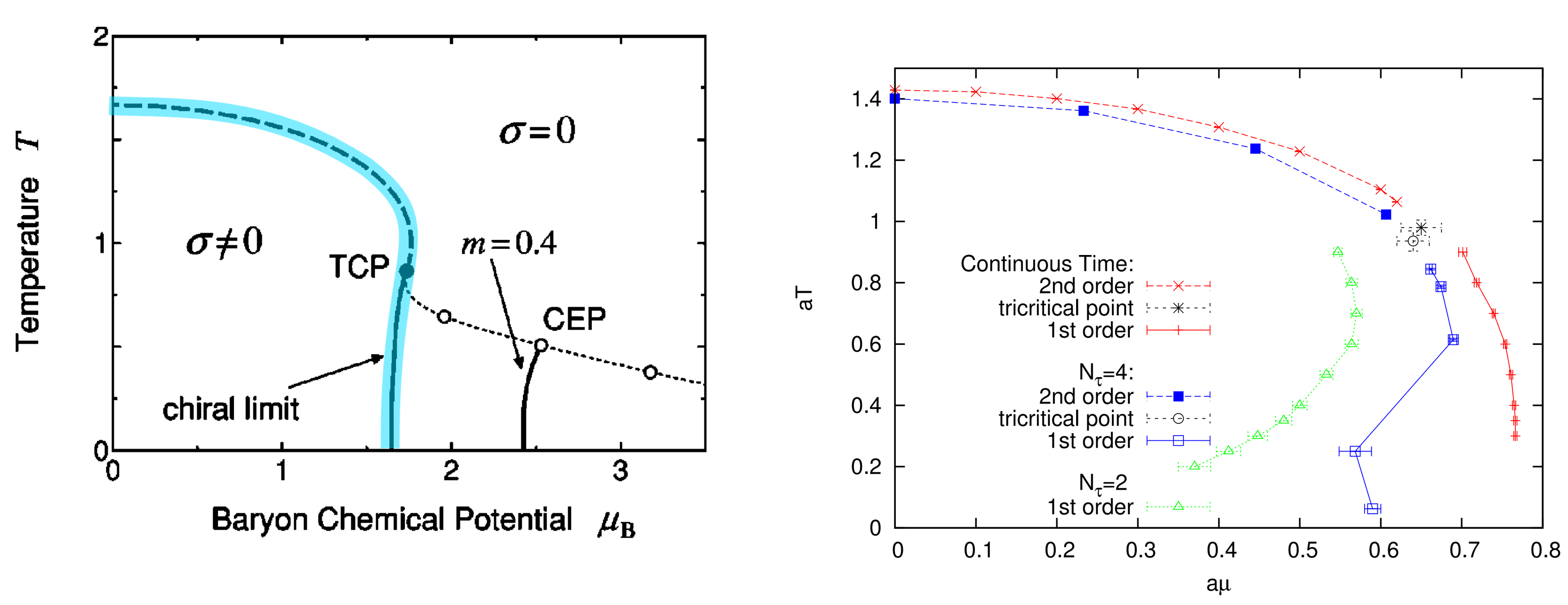


Fig. 1: SC phase diagram, left: via Mean field [2], right: from Worm algorithm [3,4,5] with identifications: $aT = \frac{\gamma^2}{N_\tau}$, $a\mu = \gamma^2 a_\tau \mu$.

Scenarios for the extension of the SC-LQCD Phase Diagram to finite β:

- back plane: strong coupling phase diagram
- front plane: continuum phase diagram ($N_f = 4$)

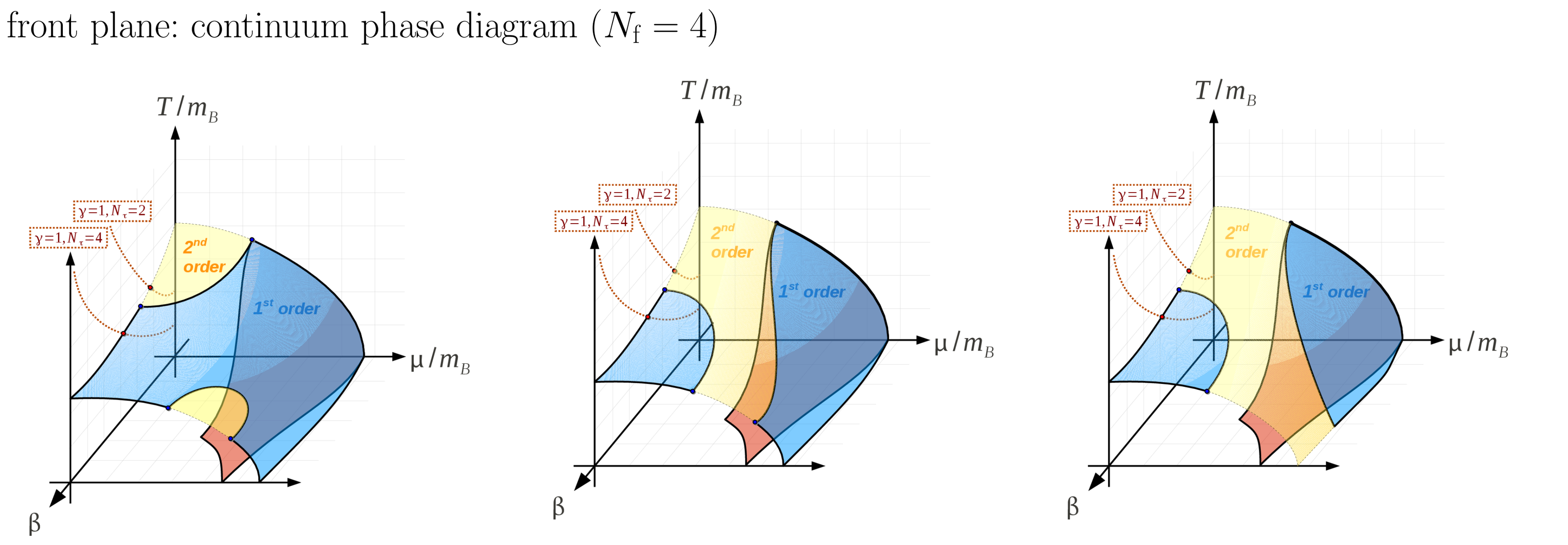


Fig. 2: Some possible scenarios: Left: two disjoint second order surfaces, middle: one second order surface, right: high density first order surface terminates at weaker coupling. The chiral and nuclear transition coincide in the strong coupling limit due to baryon saturation, but is expected to split at weaker coupling.

Questions we want to address:

- does the **tricritical point** move to smaller or larger μ as β is increased?
- do the **nuclear and chiral transition split?**

Partition function including Gauge Corrections

- Full partition function including gauge action linearized in β to obtain corrections to SC-limit:

$$Z = \int d\chi d\bar{\chi} dU e^{S_G + S_F} = \int d\chi d\bar{\chi} Z_F \left\langle e^{-S_G}\right\rangle_U \approx \int d\chi d\bar{\chi} Z_F (1 - \beta \langle S_G \rangle_U), \qquad Z_F = \int dU e^{-S_F}$$

- plaquette expectation value before Grassmann integration:

$$\left\langle \mathrm{tr}[U_P + U_P^\dagger] \right\rangle_U = \frac{1}{Z_F}\int dU\, \mathrm{tr}[U_P + U_P^\dagger] e^{-S_F} = \left(\prod_{l \in P} z_l\right)^{-1} \sum_{s=1}^{19} F_P^s(M,B,\bar{B})$$

- One-Link integrals for links on the edge of an elementary plaquette [6]:

$$J_{ik} = \underbrace{\frac{1}{3}\bar{\chi}_k\varphi_i}_{D_1} - \underbrace{\frac{1}{6}M_\chi M_\varphi \bar{\chi}_k \varphi_i}_{D_2} + \underbrace{\frac{1}{12}M_\chi^2 M_\varphi^2 \bar{\chi}_k\varphi_i}_{D_3} + \underbrace{\frac{1}{12}\epsilon_{ii_1i_2}\epsilon_{kk_1k_2}\bar{\varphi}_{i_1}\bar{\varphi}_{i_2}\chi_{k_1}\chi_{k_2}}_{B_1} + \underbrace{\frac{1}{32}\epsilon_{ii_1i_2}\epsilon_{kk_1k_2}M_\chi M_\varphi \bar{\varphi}_{i_1}\bar{\varphi}_{i_2}\chi_{k_1}\chi_{k_2} + \frac{7}{24}\bar{B}_\varphi B_\chi \bar{\chi}_k \phi_i + \frac{1}{48}\epsilon_{ii_1i_2}M_\varphi B_\chi \bar{\varphi}_{i_1}\bar{\varphi}_{i_2}\bar{\chi}_k + \frac{1}{48}\epsilon_{kk_1k_2}M_\chi \bar{B}_\varphi \chi_{k_1}\chi_{k_2}\varphi_i}_{B2}$$

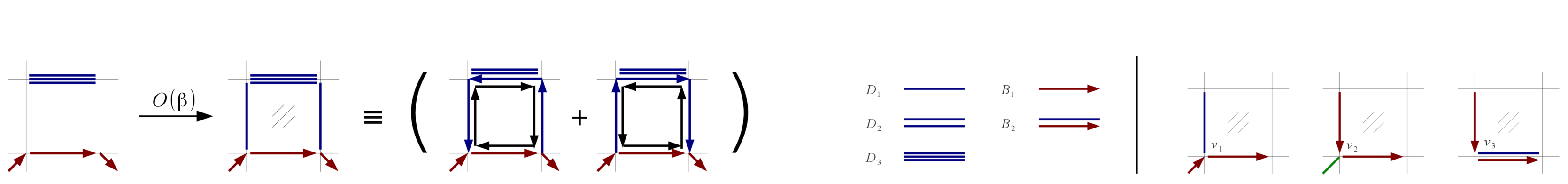


Fig. 3: Left: Graphical representation of a typical diagram at $\mathcal{O}(\beta)$. Right: Definition of link states and site states relevant for the gauge corrections.

- determine plaquette link product $P = \mathrm{Tr}\, J_{ik}J_{kl}J_{lm}J_{mi}$
- result can be consistently re-expressed via **link weights:** $w(D_k) = \frac{(N_c-k)!}{N_c!(k-1)!}$, $\quad w(B_1) = \frac{1}{N_c!(N_c-1)!}$, $\quad w(B_2) = \frac{(N_c-1)!}{N_c!}$ and **site weights:** $v_1 = N_c!$, $\quad v_2 = (N_c-1)!$, $\quad v_3 = 1$
- Grassman constraint on sites touching a plaquette is altered: $N_c \to N_c + 1$

Gauge Observables at zero and non-zero Density

Polyakov loop $\langle L \rangle$ and plaquettes $\langle P_s \rangle$, $\langle P_t \rangle$ measured via **reweighting from the SC-ensemble**:

- $\langle L \rangle = \frac{\int d\bar{\chi} d\chi \langle L \rangle_U Z_F}{\int d\bar{\chi} d\chi Z_F}$ and $\langle P_t \rangle$ are sensitive to the chiral transition
- scan at **finite density** in polar coordinates $(aT, a\mu) \mapsto (\rho, \phi)$ across the phase boundary

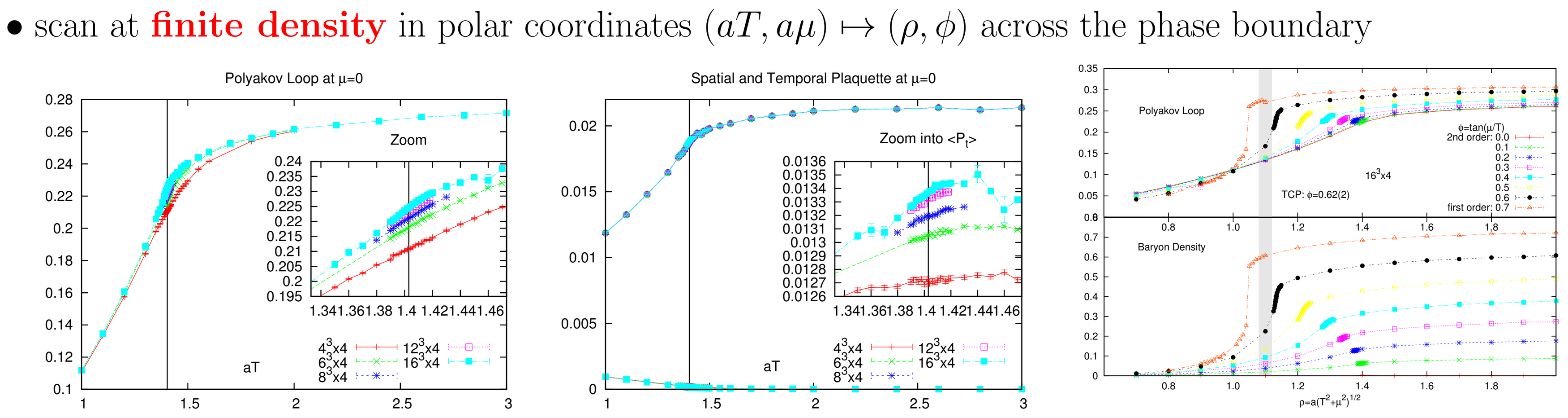


Fig. 4: Left: Temperature dependence of $\langle L \rangle$ and Plaquettes $\langle P_s \rangle$, $\langle P_t \rangle$. Right: Comparison of baryon number density with $\langle L \rangle$.

Gauge Corrections to the SC-LQCD Phase Diagram

For **fermionic observables**, e.g. the chiral susceptibility, the leading order β correction can be measured:

- obtain the slope of the transition temperature w.r.t. β from a **Taylor coefficient**:

$$\chi(\beta) = \chi_0 + \beta c_\chi^{(1)} + \mathcal{O}(\beta^2) \qquad c_\chi^{(1)} = \frac{\partial}{\partial\beta}\frac{Z_2(\beta)}{Z(\beta)}\bigg|_{\beta=0} = \langle(\bar{\psi}\psi)^2 P\rangle - \langle(\bar{\psi}\psi)^2\rangle\langle P \rangle$$

- $\chi_0 = \frac{Z_2}{Z}$ with Z_2 the 2-monomer sector sampled by $G(x_1,x_2)$ via Worm estimator,
- $c_\chi^{(1)}$ needs to obey **finite size scaling** with 3d O(2) critical exponents to modify aT_c
- one can show that in the thermodynamic limit: $\quad c_\chi^{(1)} \simeq (c_1 + c_2 L^{1/\nu} + c_3 t)$ in the vicinity of $t = 0$
- the **shift in T_c** is then related to scaling function parameters A, B and c_2: $\quad \Delta aT_c(\beta) \doteq -\beta a T_c \frac{A}{B} c_2$

Results on the Slope at Zero and non-Zero Density:

- We obtain for the **slope**: $\frac{\partial}{\partial\beta}aT_c(\beta) \simeq -0.24(3)$ at $\mu = 0$ and $\simeq -0.15(2)$ at $\mu/T = 0.29$.
- The slope vanishes at the tricritial point and along the first order line.

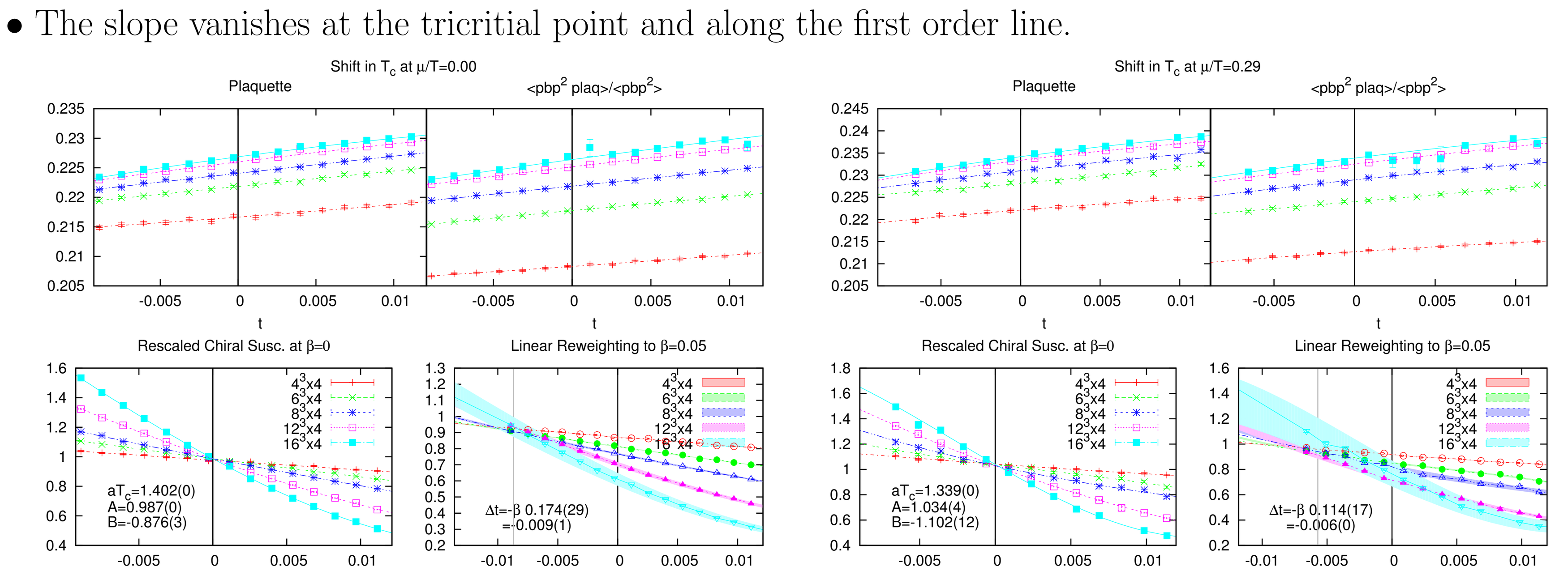
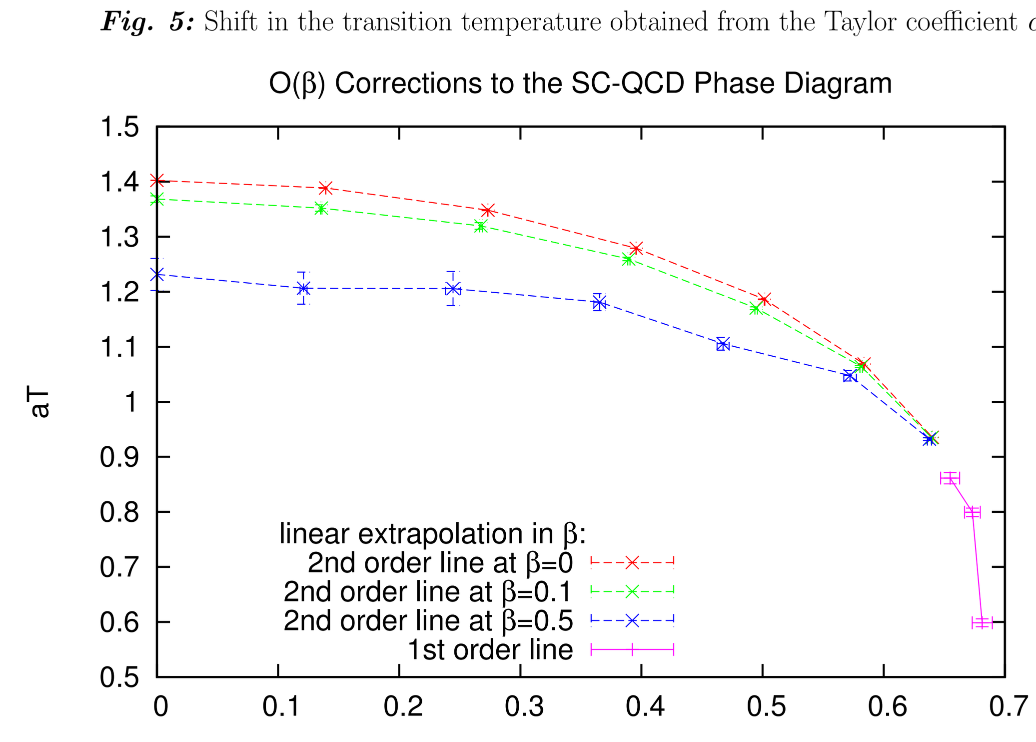


Fig. 5: Shift in the transition temperature obtained from the Taylor coefficient $c_\chi^{(1)}$ at $\mu = 0$ (left) and $\mu/T = 0.29$ (right).



- ratio at strong coupling $\frac{T_c(\mu=0)}{3\mu_c(T=0)} \approx \frac{1.403}{0.57} = 0.82$ too large compared to the continuum result in the chiral limit: $\approx \frac{154\,\mathrm{MeV}}{0.93\,\mathrm{GeV}} = 0.165$
- but: $\frac{T_c(\mu=0)}{\mu_c(T=0)} \searrow \quad (\beta \nearrow)$ at leading $\mathcal{O}(\beta)$

Fig. 6: Phase boundary in the strong coupling limit and linealy extrapolated to finite β.

Conclusion & Outlook

Achievements:

- correct average plaquette and Polyakov loop reproduced at $\beta = 0$ (checked with HMC)
- all measurements **extended to finite μ**
- $\langle L \rangle$ and $\langle P_s \rangle$ are sensitive to the chiral transition
- **slope of aT_c** determined at finite density up to the tricritical point

Further Goals:

- $\mathcal{O}(\beta^2)$ **corrections** needed: determine whether the chiral and nuclear transition split at finite β

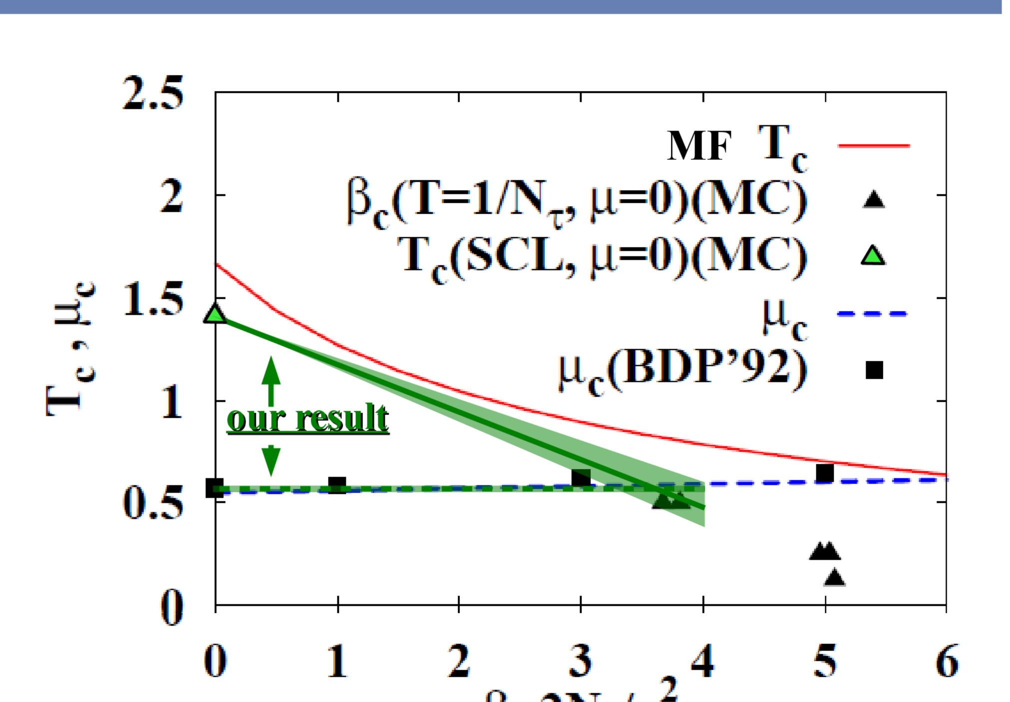


Fig. 7: Comparison of our result with mean field result by Miura *et. al* [7]: **good agreement** for the slope, extrapolates well to HMC results at large β

References

[1] P. Rossi and U. Wolff. *Nucl. Phys. B* **258** (1984) 105.
[2] Y. Nishida. *Phys. Rev. D* **69** (2004) 094501
[3] D. H. Adams and S. Chandrasekharan. *Nucl. Phys. B* **662** (1989) 220-246
[4] P. de Forcrand, M. Fromm. *Phys. Rev. Lett.* **103** (2010) 112005
[5] W. Unger and Ph. de Forcrand. hep-lat/1111.1434 (2011)
[6] M. Creutz. J. Math. Phys. **19** (1978) 2043
[7] D. H. Miura, *et al. Phys. Rev. D* **80** *(2009) 074034*