A new look at instantons at ‘large-N’

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Fujita-M.H.-Hoyos 2012(PRD)
Azeyanagi-Fujita-M.H. 2013(PRL)
Azeyanagi-M.H.-Honda-Matsuo-Shiba 2013 (hep-th+in progress)

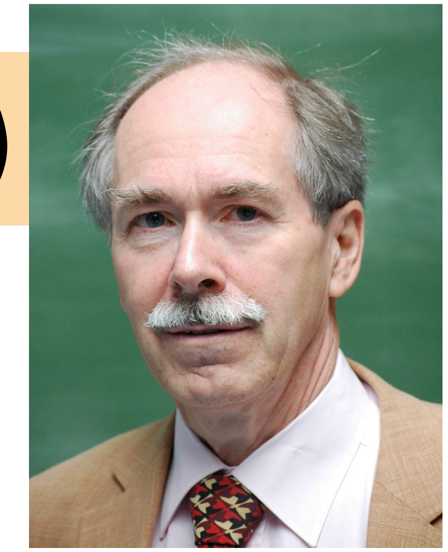
what you can learn here (introduction & summary)

- A standard folklore about the large-N limit is simply wrong. 99% of people misunderstand 't Hooft's statement.
- It turns out that nonperturbative effects can be treated straightforwardly in the large-N limit.
- Now you can easily derive full instanton partition functions of a class of non-SUSY theories.
- A conceptual difficulty for the gauge/gravity duality with the M-theory gravity dual is resolved.
- Now we have an appropriate set up to play with instantons/monopoles with 11d gravity dual -- qualitatively new applications of the gauge/gravity duality to QCD!

what is "large-N" ?

usual answer

$\lambda = g^2 N$ fixed, $N \to \infty$ ('t Hooft limit)



why?

- 't Hooft is genius.
- 1/N expansion = genus expansion. (string theory!)

$$F = \sum_{g=0}^{\infty} F_g(\lambda)/N^{2g-2}$$

- perturbative series may have a finite radius of convergence at large-N → analytic continuation to strong coupling ?
- Various nice properties (factorization, integrability,..)

classical gravity = planar limit

$$F = \sum_{g=0}^{\infty} F_g(\lambda)/N^{2g-2}$$

In AdS/CFT,

1/N correction = g_s correction	
1/λ correction = α' correction	

But what about M-theory?

$(g_{YM})^2 \sim 1$

Let's consider Another large-N limit:

$g^2 \sim N^{-\alpha}$;$\alpha=1$ is the 't Hooft limit

$\alpha<1$: 'very strongly coupled'

<u>why?</u>

- It is possible.
- application to M-theory. $(g_{YM})^2 \sim 1$
- Instanton effect remains finite.

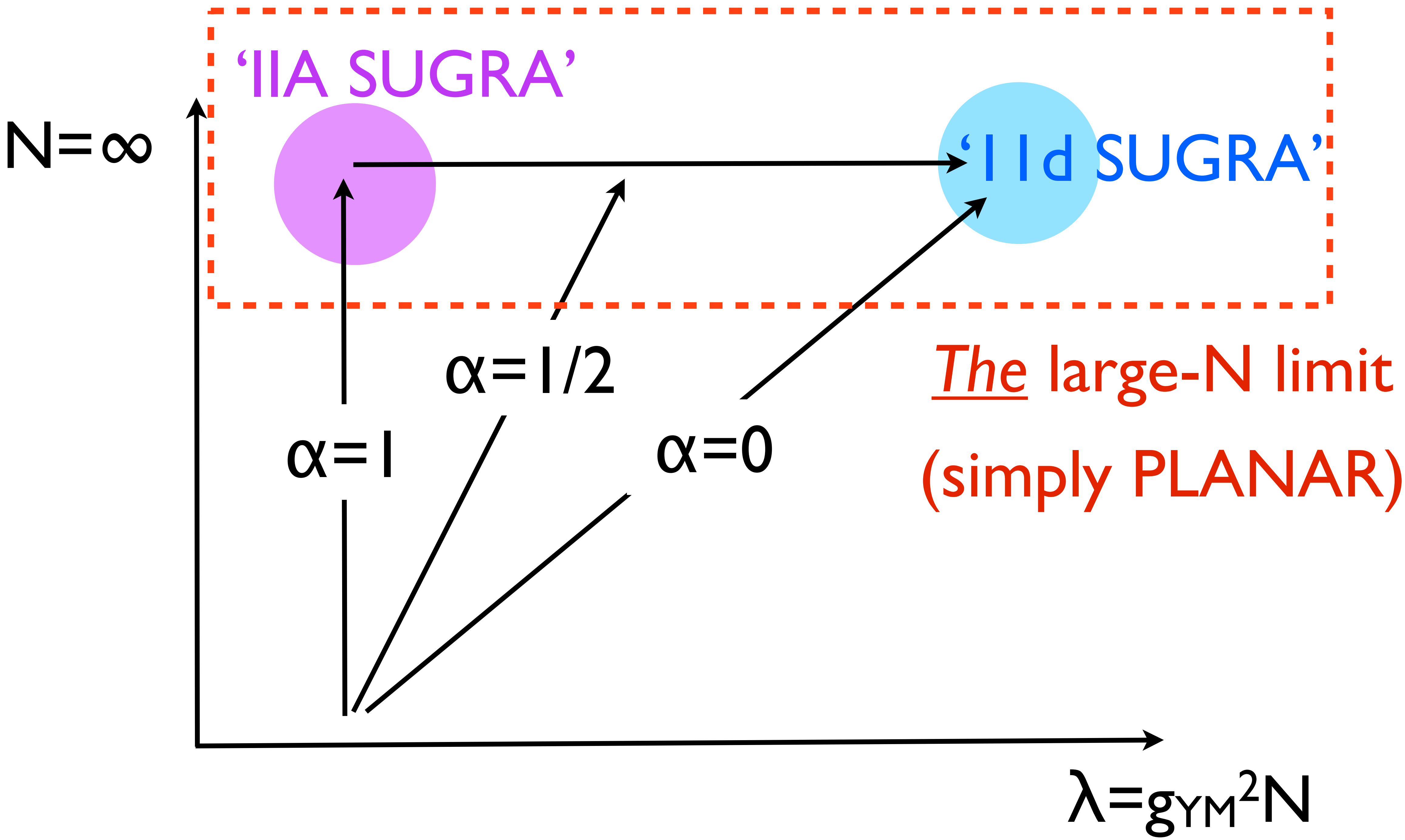
$\exp(-8\pi^2/g_{YM}{}^2) = O(1)$

λ is N-dependent.
1/N expansion and genus expansion are different.

Our conjecture

- The very strongly coupled large-N limit is well-defined and essentially the same as the ‘t Hooft limit.
- More precisely: large-N limit and strong coupling limit commute.
- When there is no ‘phase transition’ (or as long as one considers the same point in the moduli space), the analytic continuation from the planar limit gives the right answer.

(Azeyanagi-Fujita-M.H., Phys.Rev.Lett. 110 (2013))



(Azeyanagi-Fujita-M.H., Phys.Rev.Lett. 110 (2013))

A typical (wrong) objection

- Planar and nonplanar diagrams are mixed in such a limit!

$$\begin{aligned}
F(\lambda, N) &= N^2 F_0(\lambda) + F_1(\lambda) + F_2(\lambda)/N^2 + \cdots \\
&= N^2 \left(f_{0,0} + f_{0,1}\lambda + f_{0,2}\lambda^2 + \cdots\right) \\
&\quad + \left(f_{1,0} + f_{1,1}\lambda + f_{1,2}\lambda^2 + \cdots\right) \\
&\quad + N^{-2} \left(f_{2,0} + f_{2,1}\lambda + f_{2,2}\lambda^2 + \cdots\right) + \cdots \\
&= N^2 \left(f_{0,0} + f_{0,1}N + f_{0,2}N^2 + \cdots\right) \\
&\quad + \left(f_{1,0} + f_{1,1}N + f_{1,2}N^2 + \cdots\right) \\
&\quad + N^{-2} \left(f_{2,0} + f_{2,1}N + f_{2,2}N^2 + \cdots\right) + \cdots
\end{aligned}$$

Nonplanar diagrams contribute as well,
so the limits do not commute!

Why are you using a perturbative
expression at strong coupling???????

The right picture

[Azeyanagi-Fujita-M.H., Phys.Rev.Lett. 110 (2013)]

When there is a gravity dual:

$$\begin{aligned} F(\lambda, N) &= F(g_s, \alpha') \\ &= g_s^{-2} F_0(\alpha') + F_1(\alpha') + g_s^2 F_2(\alpha') + \cdots \\ &= g_s^{-2}\left(f_{0,0} + f_{0,1}\alpha' + f_{0,2}\alpha'^2 + \cdots\right) \\ &\quad + \left(f_{1,0} + f_{1,1}\alpha' + f_{1,2}\alpha'^2 + \cdots\right) \\ &\quad + g_s^2\left(f_{2,0} + f_{2,1}\alpha' + f_{2,2}\alpha'^2 + \cdots\right) + \cdots \end{aligned}$$

($g_s \sim g_{YM}^2 \sim \lambda/N$, $\alpha' \sim \lambda^{-1/2}$ for 4d N=4 SYM)

$f_{0,0}$ dominates as long as $g_{YM}^2 \ll 1$ and $\lambda \gg 1$

The same expression at $1 \ll \lambda \ll N$, simply supergravity!

(By using the S-dual, we can show it even at $N < \lambda$.)

More evidence

[Fujita-M.H.-Hoyos 2012 (PRD), Azeyanagi-Fujita-M.H. 2013 (PRL)]

- All field theories with gravity duals
- 2d pure Yang-Mills (solvable thanks to Migdal)
- strong coupling expansion of the lattice gauge theory
- Planar equivalence outside the 't Hooft limit
- Various matrix models

SUSY and/or gravity dual are not needed.

instantons

(Azeyanagi-M.H.-Honda-Matsuo-Shiba, `1307.0809 [hep-th]`)

- The same argument is valid at each instanton sector.
- Therefore 'planar dominance' holds there.
- Can be confirmed in various theories with N=2 SUSY by using the Nekrasov formula for the partition functions.

Nice properties in the planar limit holds where the instanton weight is finite!

$\exp(-8\pi^2/g_{YM}^2) = O(1)$

orbifold equivalence between instanton patition functions

(※ It is just one of various examples, which is almost trivial *from our new viewpoint*.)

'parent' → 'daughter'

4d N=2 SYM —Z_k orbifolding→ YM with less SUSY

Nekrasov formula

you can take it to be non-SUSY!

$$Z_{\text{p-inst, parent}} = \left(Z_{\text{p-inst, daughter}}\right)^k$$

in THE large-N limit

When the daughter keeps N=2 SUSY,
you can easily confirm it by using the Nekrasov formula.

(Azeyanagi-M.H.-Honda-Matsuo-Shiba, `1307.0809 [hep-th]`)

M-theoretic holography

(in progress)

- 11-d SUGRA should know the instantons, monopoles, etc at large-N with g^2 fixed. This is nothing but 'planar' in the gauge theory side.
- It should be possible to study the dynamics of solitons by using 11d SUGRA!
- As a first nontrivial test:

On-shell action of 11d SUGRA (Gaiotto-Maldacena geometry)	=	Free energy of 4d N=2 gauge theory (Gaiotto theory)

(conjecture; now checking it.)

(non-QCD) Application: M5 from 5d SYM

[Azeyanagi-Fujita-M.H., Phys.Rev.Lett. 110 (2013)]

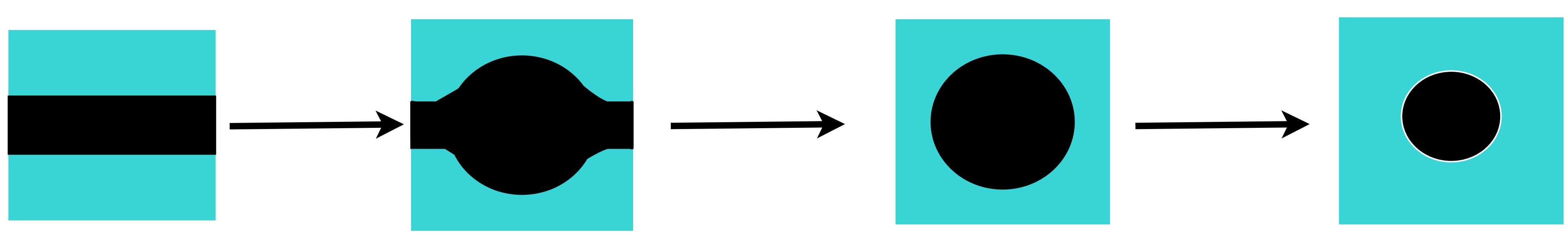
D0-branes in $R^{1,9}$ = 1d SYM (BFSS matrix model)
(black hole)

↓

'black string' winding on M-circle (+ boost)

↓ <u>'Gregory-Laflamme transition'</u>
Analytic continuation doesn't work!

11d black hole localized along M-circle (+ boost)



(Itzhaki-Maldacena-Sonnenschein-Yankielowicz 1998)

D4-branes in $R^{1,9}$ = 5d SYM

↓

<u>no singularity</u>

Analytic continuation can work!

M5-branes winding along M-circle

↓

M5-branes in $R^{1,10}$

This region is conjectured to be described by 5d SYM

<u>5d SYM = 6d field theory on M5</u>

(Douglas 2010; Lambert, Papageorgakis, Schmidt-Sommerfeld 2010)

We can test this conjecture at large-N.

5d SYM vs M5

- Calculate quantities in 5d SYM in the planar limit, by using the gravity dual (black 4-brane in IIA SUGRA).
- Then analytically continue them to $g_{YM} \sim 1$ and compare to M5-branes.
- Free energy of the finite-temperature M5 is reproduced from 5d SYM.

$$F_{D4} = c\underline{\lambda N^2} T^6 V_4 = F_{M5}$$

$$\underline{\lambda N^2} \rightarrow R_M \times N^3$$

- Two-point function and entanglement entropy are also reproduced.